Kurt Gödel Day

KGD2024

&

Czech Gathering of Logicians

CGL2024

27–28 May 2024
Brno, the Czech Republic

Book of abstracts

Contents

Locations

Conference

– Mon 27/5/2024 – Tue 28/5/2024

see website (below) for maps, schemes, photos

– Mon: Brno Observatory and Planetarium, Kraví hora 522/2, Brno

– Tue: Faculty of Arts, Masaryk University, Arne Nováka 1, Brno; B2.23

Conference lunch

– Mon 27/5/2024, 12:30

– Brno Observatory and Planetarium, Kraví hora 522/2, Brno

main building

Conference banquet

– Tue 28/5/2024, 19:00

– Brno Observatory and Planetarium, Kraví hora 522/2, Brno

main building

Website

`https://www.physics.muni.cz/~godel/kgd2024/`

Sponsors

Brno Observatory and Planetarium

Department of Philosophy, Masaryk University

Kurt Gödel Society in Brno

Czech Society for Cybernetics and Informatics

Call for papers

Czech Gathering of Logicians and Kurt Gödel Day 2024 aim at bringing together researchers in all areas of logic. While the event usually attracts mainly logicians based in the Czech Republic, researchers working in other countries are warmly welcome as well. The conference language is English. Contributions related to Gödel's work are especially welcome, continuing the tradition of the Kurt Gödel Society in Brno events.

This year's Gathering is organized by Masaryk University and co-organized by Brno Observatory and Planetarium, Kurt Gödel Society in Brno, Institute of Computer Science of the Czech Academy of Sciences, and the Union of Czech Mathematicians and Physicists, Brno branch.

Researchers working in a field relevant to the conference are encouraged to submit an *abstract of 1–2 pages*, including references. New and recent research work is welcome for presentation.

Authors should prepare their abstracts using the *EasyChair LaTeX style*, available at `https://easychair.org/publications/for_authors` (non-LaTeX users may use EasyChair MS Word template).

All submissions will be evaluated by the *programme committee*. Accepted submissions will be presented within the contributed talk sessions, in slots lasting approximately 20 to 30 min. The conference language is English, both for submission and for presentation.

Deadline for paper submission: April 22 2024 (extended).

Notification of acceptance: May 7 June 2024.

Conference fee (includes booklet of abstracts, coffee breaks and conference dinner): CZK 2.000 [2.500 late] (for students: CZK 400).

Registration of speakers required.

Programme and organizing committees

Programme committee

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Organisation

Masaryk University
Brno Observatory and Planetarium
Kurt Gödel Society in Brno
Institute of Computer Science, Academy of Sciences

Programme

Monday 27 May 2024

9:30	*opening* at Planetarium	
10:00	**beginning** of talks	
	invited talk	*chair*: Jiří Raclavský
10:00	Andrzej Indrzejczak	Proof Systems for Hybrid Logic The with Lambda and Iota Operators
11:00	*coffee break*	
	contributed talks	*chair*:
11:30	Libor Běhounek	On the Lottery-style Paradoxes in Positive Free Logics
12:00	Roman Kuznets	A priori Knowledge in Distributed Systems
12:30	**lunch**	at the site (courtesy of Planetarium)

	invited talk	*chair*:
14:00	Rostislav Horčík	A Logical Approach to Explainability for Graph Neural Network
15:00	*coffee break*	
	contributed talks	*chair:*
15:30	Thomas Ferguson and Jitka Kadlečíková	Variations on Monstrous Content
16:00	Igor Sedlár	Algebras for Relevant Reasoners
16:30	*coffee break*	
	contributed talks	*chair:*
17:00	Zuzana Rybaříková	Logical Judgement vs. Sentence vs. Proposition: Formulation of Polish Logical Terminology
17:30	Karel Šebela	Theory of Concepts and Intensional Interpretation of Aristotelian Logic
18:00	**end** of the first day	

Tuesday 28 May 2024

9:00	*opening* at Faculty of Arts	
	invited talk	*chair*:
09:30	Amirhossein Akbar Tabatabai	On Gödel's Classical Interpretation of Intuitionism
10:30	*coffee break*	
	contributed talks	*chair:*
11:00	Igor Sedlár, Ondrej Majer and Chun-Yu Lin	A Logic of Probability Dynamics
11:30	Kateřina Trlifajová	Philosophical Reasoning of the Alternative Set Theory
12:00	Timotej Šujan	Structural Differences of Paradoxes of Self-reference
12:30	**lunch**	in nearby plentiful restaurants, e.g.
		Garden Food Concept - Kounicova 284/39 Mediteran Bistro - Smetanova 1022/19 Restaurace Zdravý život [veggie] - Jaselská 194/11

	invited talk	*chair*:
14:00	Jiří ROSICKÝ	Stable Independence from the Category Theoretic Point of View
15:00	*coffee break*	
	contributed talks	*chair:*
15:30	Alena VENCOVSKÁ	Strongest Principles of Pure Inductive Logic
16:00	Raheleh JALALI	Is Every Interpolation Procedure Complete?
16:30	Filip JANKOVEC, Petr CINTULA and Carles NOGUERA	Łukasiewicz Unbound Logic and its Completeness Theorem
17:00	**end** of talks	
18:30	*coffee* at Planetarium	
19:00	**conference banquet**	
19:00	KURT GÖDEL PRIZE	ceremony for prof. Jiří ROSICKÝ
19:15	*chamber music*	*Trio komorní dechové harmonie Brno* (Chamber Wind Harmony Trio Brno)* Leoš *Janáček* – from The Overgrown Path Scott *Joplin* – selected ragtimes from Portrait

* Zdeněk MIKULÁŠEK (clarinet), Jaroslav STRMISKA (clarinet), Natálie KHEMLOVÁ (bassoon)

I. Invited speakers

Rostislav Horčík

A Logical Approach to Explainability for Graph Neural Networks

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Graph Neural Networks (GNNs) are increasingly becoming popular machine learning models for tackling tasks that involve analyzing complex structured data representable as graphs or, more precisely, as binary relational structures. With the rapid evolution of different GNN architectures, there's a growing effort within the machine learning community to develop techniques that can explain the predictions made for a given input graph G. The prevalent strategies aim to identify a subgraph S of G deemed "significant" for the prediction, with its size being constrained. This significance is typically assessed using a fidelity measure, where the most common metrics examine if S yields the same prediction as G and, conversely, if removing S changes G's prediction. This approach may need to be revised when considering the recent characterization of GNNs' expressiveness using C_2 logic, a subset of First-Order Logic that includes counting quantifiers and is limited to two variables. Drawing from this, I propose reconsidering what constitutes an explanation, suggesting a more "logical" perspective. Specifically, when a GNN classifies input graphs into a finite number of classes (assuming, for simplicity, two classes: 0 and 1), an ideal explanation would be a C_2-theory T. The class of graphs classified as 1 can be axiomatized using C_2-sentences. Therefore, explaining a prediction for a single graph G involves finding a C_2-sentence that is true in G and whose models also satisfy the theory T. Even though this task is theoretically solvable, it is not computationally tractable. I will close the lecture by discussing possible research directions for approximating the solution.

Andrzej Indrzejczak

Proof Systems for Hybrid Logic with Lambda and Iota Operators

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Hybrid logics provide significant and powerful extension of modal logics. In particular, they offer well-developed proof theory and suitable framework for dealing with natural languages. One of the specific features of natural languages is common use of complex names conveying information about their intended designates. Definite descriptions, usually formalised by means of iota operator, are of special interest, and as such they were intensely investigated by philosophers of language. Since particularly interesting and difficult problems are generated by their behaviour in intensional contexts, it is important to provide their suitable formal treatment in modal logic. We focus on the development of well-behaved proof theoretic tools for dealing with the iota (for complex terms) and lambda (for complex predicates) operator in hybrid logic. Two such proof systems will be discussed. The first is was developed first by Fitting and Mendelsohn in the setting of standard modal logic, and then reformulated by Indrzejczak in the setting of hybrid logic. The second was developed recently by Indrzejczak and Zawidzki on top of hybrid tableau system of Blackburn and Marx.

Jiří Rosický

Kurt Gödel Society in Brno Award Recipient

Stable Independence from the Category Theoretic Point of View

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Amirhossein Akbar Tabatabai

On Gödel's Classical Interpretation of Intuitionism

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In 1933, Gödel introduced a provability interpretation for intuitionistic propositional logic, IPC, reading the intuitionistic truth as the classical provability [1]. In his interpretation, instead of using any concrete classical proof, he employed the modal system S4 as a formalization for the intuitive concept of provability and translated IPC into S4 in a sound and complete manner. This clever use of S4 ties the system IPC to the classical provability, as he imagined. However, it also leaves us wondering if there is any concrete provability interpretation for the modal logic S4 and hence for the intuitionistic truth.

In this talk, we provide this missing provability interpretation. We first generalize Solovay's seminal provability interpretation of the modal logic GL to capture other modal logics such as K4, KD4 and especially S4. The main idea is introducing a hierarchy of arithmetical theories to represent the informal hierarchy of theories, meta-theories, meta-meta-theories and so on and then interpreting nested modalities as the provability predicates of different layers of this hierarchy. The interpretation is always fixed and different conditions on the hierarchy correspond to different modal logics the interpretation captures.

Then, in the second part, we combine our provability interpretation for modal logics with Gödel's translation to provide a concrete provability-based interpretation for some propositional logics. The interpretation and its dependency on the hierarchy of meta-theories suggests that the term "intuitionistic logic" is a plural name for a variety of propositional logics including intuitionistic logic, minimal logic and Visser-Ruitenburg's basic logic. They are all intuitionistic logics, believing in truth as the classical provability. Their differences, however, stem from the different ontological commitments they put on their hierarchy of meta-theories.

[1] K. Gödel, Eine Interpretation des Intuitionistichen Aussagenkalküls, *Ergebnisse Math Colloq.*, vol. 4 (1933), pp. 39-40.

II. Contributed talks

On the lottery-style paradoxes in positive free logics

Libor Běhounek

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Positive free logic [4] is a variant of predicate logic that admits non-denoting terms and where predications about non-denoting terms can be true (**t**), false (**f**), or neither true nor false (**n**). Positive free logic can be developed over various paracomplete logics, including Kleene's three-valued logic K3, Belnap–Dunn's four-valued logic BD, and their variants [1]. However, I will argue that over these logics, some predications about non-denoting terms suffer from lottery-style paradoxes, where each member of a disjunction can intuitively be considered neither true nor false (**n**), while the whole disjunction can be considered true (**t**); yet when evaluated in K3 or BD, a disjunction with all disjuncts evaluated to **n** always comes out as **n**.

The arguments in [3, §2.4] suggest that in these cases, Belnap would advocate valuing the whole disjunction by **n**. In the talk, I will counter-argue that in positive free logic, evaluating such disjunctions by **t** can be well justified, and that the resulting non-truth-functionality can be resolved by using a three- or four-valued modality, which captures Belnap's informal interpretation of the four truth values of the logic BD in terms of 'being told true' and (independently) 'being told false' [2].

Acknowledgments: Supported by grant No. 22-01137S of the Czech Science Foundation. Partly based on a joint work with Martina Daňková and Antonín Dvořák.

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[2] N. Belnap. How a computer should think. In G. Ryle, editor, *Contemporary Aspects of Philosophy.* Oriel Press, 1977.

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Łukasiewicz unbound logic and its completeness theorem

Petr Cintula[1], Filip Jankovec[1], and Carles Noguera[2]

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In this talk, we investigate connections between the family of comparative logics including Abelian logic, and some generalizations of Łukasiewicz logic.

Łukasiewicz logic in its infinitely-valued version was introduced by Łukasiewicz and Tarski [9] in 1930 and since then it proved to be one of the most prominent non-classical logics. This logic is by itself a member of the family of many-valued logics often used to model some aspects of vagueness. Also, it has deep connections with other areas of mathematics such as continuous model theory, error-correcting codes, geometry, algebraic probability theory, etc. [3, 7, 6, 11].

Unbound Łukasiewicz logic introduced in [4] is a generalization of Łukasiewicz logic. Apart from the philosophical and linguistic motivations, this logic can also be motivated purely syntactically, namely, the connectives of unbound Łukasiewicz logic can be seen as an untruncated version of connectives of standard Łukasiewicz logic.

Abelian logic is a well-known contraclassical paraconsistent logic. This logic was independently introduced by Meyer and Slaney [10] and by Casari [2] and it is also called the logic of Abelian ℓ-groups [1] or Abelian Group Logic [12]. This terminology follows from the fact that the matrix models of Abelian logic consist of Abelian ℓ-groups and their positive cones as filters of designated elements (there is also a version of Abelian logic in which the only designated element is the neutral element of the group, which will not be considered here).

Recall that in Abelian logic the constant 0 plays the role of the valid statement (we have $0 \leftrightarrow (\varphi \leftrightarrow \varphi)$ for each formula φ) and also the false statement ($\neg\varphi \leftrightarrow (\varphi \to 0)$ for each formula φ). We would like to construct a logic that allows us to separate these two meanings. To achieve this goal we add to our language a new constant symbol f and we redefine the negation in the following way: $\neg\varphi := (\varphi \to f)$, in particular $\neg 0 := f$. We call this logic the pointed Abelian logic.

In this talk we are interested in logics which are semilinear, i.e. they are determined by their totally ordered models. By additional axioms, we may force realizations of 0 and f to appear in a specific order. For example, we show that the models of pointed Abelian logic where f is interpreted as strictly smaller then 0 corresponds to the models of unbound Łukasiewicz logic.

We provide axiomatizations for these logics and we discuss their finite strong completeness properties, in which we use the characterizations from [5], and their possible philosophical interpretations.

This talk is based on the joined work with Petr Cintula and Carles Noguera.

References

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Variations on Monstrous Content

Thomas M. Ferguson[1] and Jitka Kadlečíková[2,3]

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In [5], Restall introduced a *bounds consequence* interpretation of sequent calculi in which provability of a sequent $[\Gamma \succ \Delta]$ (where Γ and Δ are multisets of formulae) indicates that to assert each of Γ while denying each of Δ violates conversational norms. *E.g.*, the provability of $[\varphi \succ \varphi]$ corresponds to a proscription against both asserting and denying a sentence in a particular context. This was taken up and refined by Cobreros, Egré, Ripley, and van Rooij in [2], in which the bounds consequence reading gives rise to *strict-tolerant* logic ST with the truth predicate. On this view, the sequent $[\Gamma \succ \Delta]$ is understood as the impossibility of each of Γ being strictly true (*i.e.* "only true") while each of Δ is strictly false (*i.e.* "only false").

Consequently, the sequent $[p \succ p]$ can be ruled out as having violated *veridical bounds* on conversational positions; a position in which one asserts p while denying p cannot be tenable as one thereby *blows hot and cold*. In particular, the inclusion of the third value of the strong Kleene matrices receives an interpretation as a *veridical defect* on the reading of Cobreros *et al.*, ruling out the assertion or denial of the liar sentence λ.

But veridical proscriptions against positions are but one dimension of the myriad distinct constraints over conversational bounds. Recently [3] offered an interpretation acknowledging the influence that *topic-theoretic considerations*—rather than veridical concerns—exert over the shape of the bounds. ST and its extension iST allow for a rigorous analysis of such topic-theoretically determined bounds. These systems are determined by taking the strict-tolerant interpretation of the weak Kleene matrices, in which the truth value $\frac{1}{2}$ is assigned the role of standing in for a topic-theoretic defect, *e.g.*, language including slurring, blasphemies, or other sentences ruled out prior to the evaluation of their veridical status. A position, then, can be ruled out *in a particular context* in virtue of its topic-theoretic assumptions. In [3], such topic-theoretic defects are described as *monstrous content*.

Of course, there are further subtleties concerning how such monstrous content ought to be modelled. Two clusters of such concerns naturally emerge: First, how the semantic behavior of topic-theoretic defects can be complicated, and second, how the semantics of the interaction between topic-theoretically and veridically defective sentences plays out. In this piece, we will discuss several modifications to the picture of [3] and describe some formal results concerning them.

In the first cluster, we can describe two complications for the particular semantic model of monstrous content:

- One can think that some topic-theoretic transgressions are "worse" than others in particular contexts. A given situation—say, a dinner party—may rule particular assertions out-of-bounds by degrees; one topic may be mildly irritating while another may require gross censure. If we are to distinguish different *degrees* of topic-theoretic defects, we can do so by replacing the single value $\frac{1}{2}$ with a dense linear ordering of values e_i; a matrix semantics following this intuition can be defined that will be referred to as the fm ("fuzzy monstrosities") matrices.

- One may also take note of the fact that the topic-theoretic transgressions associated with particular complex expressions—say, a conjunction—are not located in any subexpressions in isolation, but only in their concatenation. Many *complex stereotypes* involve several ascriptions that are offensive together but anodyne individually. In such cases, the semantic behavior of such *emergent* phenomena are particularly well-modelled following the idiom of non-deterministic matrices ("Nmatrices") introduced in [1]. We will present such an Nmatrix approach, referring to them as the em ("emergent monstrosities") matrices.

In the second cluster, we can reduce the question to the matter of reconciling the interaction between a veridically defective sentence (like λ, the liar sentence) and a topic-theoretically defective sentence (like @#$%&!, or grawlix, *i.e.*, a sentence containing offensive content). Considering how to treat a sentence $\lambda \wedge$ (@#$%&! $\vee \neg$@#$%&!) reduces down to the matter of whether this sentence is ruled unassertable for being *untrue* or for being *offensive*. Two approaches to this can be described:

- One could consider that the interaction of topic-theoretic and veridical defects should defer to topic-theoretic considerations. *E.g.*, if one accepts that an obligation to speak without causing gross offense is prior to an obligation to speak truthfully, then a position [$\lambda \wedge$ (@#$%&! $\vee \neg$@#$%&!) $\succ$] is out-of-bounds on topic-theoretic grounds. This deferral to topic-theoretic priority induces a set of matrices we can describe as the ds ("deterministic synthesis") matrices.
- One could alternatively remain agnostic about the matter, allowing such corner cases to non-deterministically choose between two types of defect. Again, borrowing from the techniques of Lev and Avron in [1], such a position would require that the value assigned to $\lambda \wedge$ (@#$%&! $\vee \neg$@#$%&!) cannot be determined *a priori* but must at least be selected between the two types of defects. This agnostic posture induces matrices that we call the nds ("non-deterministic synthesis") matrices.

To conclude, we will appeal to several recent results described in [4] concerning a property of *counterexample sufficiency* concerning translations between many-valued matrices and preservation of valid sequents in a strict-tolerant setting. This will show a type of entrenchment or invariance according to which many natural variations on monstrous content do not affect the consequence relation (though, notably, the set of admissible rules may differ) and that when values for veridical defects are added, ST is extremely entrenched. Formally, these results are that:

Observation 1. *A sequent* $[\Gamma \succ \Delta]$ *is valid on the strict-tolerant readings of either the* fm *or* em *matrices precisely when* $[\Gamma \succ \Delta]$ *is* iST *valid.*

Observation 2. *A sequent* $[\Gamma \succ \Delta]$ *is valid on the strict-tolerant readings of either the* ds *or* nds *matrices precisely when* $[\Gamma \succ \Delta]$ *is* ST *valid.*

References

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Is every interpolation procedure complete?

Raheleh Jalali

April 23, 2024

Craig interpolation is a fundamental property of logic. The question of which interpolants can be obtained from an interpolation algorithm is of profound importance. Motivated by this question, we initiate the study of completeness properties of interpolation algorithms. Suppose a calculus G for propositional logic (for instance the propositional **LK**) and an interpolation procedure $\mathcal{I}$ (for example, the Maehara-style interpolation procedure) are given. We are interested in the power of the interpolation procedure with respect to the calculus. Stating the problem precisely: Let C be a (semantically possible) interpolant for a given tautology $A \to B$. Does there exist a proof π of $A \to B$ in G such that $\mathcal{I}(\pi)$ is logically equivalent to C? A positive answer to this question allows us to call the interpolation procedure $\mathcal{I}$ *complete* for G. If we take G to be the cut-free propositional **LK**, then the standard Maehara-style interpolation (call it M) fails to provide a positive answer to the question. Similarly, for propositional resolution and the standard algorithm to find the interpolant. However, if we take G to be the propositional **LK** with atomic cuts, then M is complete for G. This shows that to construct any possible interpolant via the Maehara-style interpolation procedure, using the cut rule is inevitable. What if we move to the realm of first-order logic? Then, obviously, M is incomplete for the cut-free first-order **LK**. Interestingly though, M for first-order **LK** with atomic cuts is also incomplete. This talk is based on a joint work with Stefan Hetzl.

A priori Knowledge in Distributed Systems*

Roman Kuznets

TU Wien

In epistemic modal logic [6], common knowledge of the model is necessary to explain higher-order reasoning of agents [1]. The same principle, when transferred to runs and systems framework, is routinely used to reason about knowledge in distributed systems [5]. In other words, to explain what agents know, what agents know about other agents, and how they are able to interpret messages from other agents, it is necessary to assume that they have a priori common knowledge of all possible runs of the system and of the protocols of all agents, including their communication protocols. This high level of assumed a priori knowledge is usually implicit, which may explain why it stayed largely under the radar of epistemic logical analysis.

While convenient in its simplicity, this assumption also leads to a certain inflexibility. When faced with a situation not predicted a priori, agents cannot perform any action. One suggestion for overcoming this problem would be to pre-program all possible scenarios. However, for complex systems, this might be problematic both in terms of feasibility (too many exceptions to consider) and in terms of completeness (it is not always possible to guarantee that all exceptions are taken into account). It is, of course, possible to pre-program agents not to react in any way when faced with an exception. But that risks them crashing if the exception is not transient and does not lift on its own.

The ability to self-recover and to adapt one's behavior is especially important when considering so-called self-adaptive and self-organizing (SASO) system [2, 7].

But even in traditional distributed systems, it is quite common that the a priori "knowledge" pre-programmed by the system designer contains mistakes. It is, thus, better to talk about agent's a priori beliefs. When these beliefs are found not to be factive, e.g., as a result of testing, the system designer is expected to correct them, which amount to the operation of updating a priori beliefs [4].

One area of logic that deals with updating knowledge/beliefs is dynamic epistemic logic [8], which extends epistemic logic with update operators that model the changes in the agents' epistemic attitudes. Hence, it makes sense to explore how updates of a priori beliefs can be performed using a similar mechanism.

We consider simple epistemic puzzles, such as Muddy Children Puzzle and Consecutive Numbers Puzzle, and show how to add explicit a priori assumptions to them. We then develop a semantic update mechanism that is triggered when an agent's beliefs become inconsistent and acts by trying to modify the agent's explicit a priori beliefs to restore consistency. Because these updates are triggered privately, they necessarily violate the common knowledge of the model. We explore some consequences of this by presenting several examples of how the proposed mechanism can be used. [3]

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Logical Judgement vs. Sentence vs. Proposition: Formulation of Polish Logical Terminology

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Abstract

The talk will deal with the difference in Polish and English logical terminology, namely on the term 'proposition' in English, which appears as 'zdanie' in Polish logical terminology. It describes the development in the Polish logical terminology and focuses mainly on Jan Łukasiewicz's contribution to the development. It will also stress that there is a twofold translation of the term 'proposition' in Polish, i.e. 'zdanie' in logic and 'sąd w znaczeniu logicznym' in the philosophy of language.

1 Introduction

There is an interesting difference between the name of one of the building blocks of logic in English, Czech and Polish. While there is a propositional calculus in English, Czech logicians investigate a subject 'výrokový kalkul' instead and Polish logicians delve into 'rachunek zdań'. Since Frege, propositions have retained an extensive philosophical background (see McGrath and Frank 2020). A Czech term 'výroky' contained a more modest philosophical background. They are linked with the act of stating. Thus, Czech logicians deal with a 'calculus of statements'. In contrast, the term 'zdanie' could be translated as 'sentence'. Therefore, Woleński (1989, 97) entitled the systems of logic developed in the Polish group of logicians, the Lvov-Warsaw School as a 'sentential calculus'. My talk will focus on how this approach in Polish terminology appeared.

Peter Simons (2023) argues that it was Jan Łukasiewicz who introduced this terminology, and that the terminology was a part of Łukasiewicz's denial of psychologism. As Łukasiewicz was an important scholar in the Lvov-Warsaw School and the School influenced considerably a logical scene in Poland, Łukasiewicz's terminology spread in Polish logic.

2 The Terminology in the Lvov-Warsaw School

Already the founder of the School, Kazimierz Twardowski, held courses on mathematical logic. However, Twardowski did not share the fascination of mathematical logic that later appeared among his students. He preferred psychology as a tool of scientific philosophy (see Brożek 2022, 8–9). When he addressed a proposition, Twardowski (1901, 13–14, 18) used the term 'sąd' i.e. 'judgement', similarly to Brentano.

At first, Łukasiewicz adopted Twardowski's terminology which was also used in previous Polish logical texts. However, he (1907/1961, 64) pointed out that the term 'sąd' could obtain two different meanings, in his paper 'Psychologia a logika' [Psychology and Logic], in which he argued against psychologism in logic. Namely, he differentiated between judgements in a psychological and logical sense. Nonetheless, he later did not see this differentiation from psychology as sufficient. In 1914, he began to use the term ‘zdanie’ for propositions instead (see Rybaříková 2022, 9–10).

This change was generally adopted in the logical terminology of the Lvov-Warsaw School. Nonetheless, certain members argued for ‘judgements in the logical sense’ in the philosophy of language. Ajdukiewicz (1934, 104) and Czeżowski (1957, 20) argued for preservations of ‘judgements in the logical sense’ as meanings of sentences.

3 Conclusion

Although at the beginning of mathematical logic in Poland, propositions were translated as judgements, they are nowadays addressed as sentences. It was Łukasiewicz driven by his war against psychologism that started this change in Polish logical terminology. Judgements in the logical sense were, however, preserved as meanings of sentences in the work of certain philosophers. In that way, it could be claimed that propositions have two ways of translation in Polish. They are translated as sentences in logic and as judgements in the logical sense in the philosophy of language.

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Algebras for Relevant Reasoners

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In my recent work with Pietro Vigiani [3] we introduced a semantic framework for epistemic logic which combines classical propositional logic with relevant modal logic. The characteristic feature of the framework is that epistemic operators satisfy closure under relevant logic, but not closure under classical logic. As such, the framework is an expansion of earlier work by Levesque [1]. The framework is based on relational models and it is natural to ask what the corresponding algebraic semantics is. We answer this question in the present contribution. Our algebraic models are Boolean-ordered modal Dunn groupoids (BMD-groupoids), that is, positive Ackermann groupoids of [2] with a negation satisfying De Morgan laws and a modal box operator distributing over conjunction, extended with a Boolean preorder. A Boolean preorder on a Dunn groupoid is a preorder that induces a Boolean quotient algebra. A representation theorem for BMD-groupoids with respect to complex algebras of bounded Routley–Meyer frames with worlds [3] is established.

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A Logic of Probability Dynamics

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Building on the work of Hájek et al. [1, 2, 4] we introduce a version of (crisp) modal Łukasiewicz logic [3] suitable for formalizing reasoning about *probability dynamics*, that is, processes resulting in a change of subjective probabilities agents assign to events. In our model, processes of this kind are modelled as state transitions endowed with semiring structure.

Let K be a set of action letters and X a set of event letters. Let the sets *action expressions* Act_K and the *event formulas* Fm_X be defined as follows:

$$\alpha, \beta ::= a \in K \mid \alpha;\beta \mid \alpha \cup \beta \mid 1_K \mid 0_K \qquad e, f ::= x \in X \mid \neg e \mid e \vee f$$

(We assume the usual definitions of other Boolean operators on event formulas.) Let I be a set of agents. The set of formulas Fm (over I, K and X) is defined as follows:

$$\varphi, \psi ::= P_i(e) \mid 0 \mid \neg\varphi \mid \varphi \oplus \psi \mid [\alpha]\varphi$$

(We assume the usual definitions of other Łukasiewicz operators such as $\ominus$ and $\to$. We denote as Fm_{-K} the $[\alpha]$-free fragment of Fm.)

Definition 1. *An FP(KŁ)-model is a triple $\mathfrak{M} = \langle S, R, V \rangle$ where S is a non-empty set, R is a homomorphism from Act_K to the semiring of binary relations on S, and $V : Fm \times S \to [0,1]$ such that*

- *for all $s \in S$ and $i \in I$, $\mu_{s,i} : e \mapsto V(P_i(e), s)$ is a finitely additive probability measure (on the quotient of Fm_X by Boolean algebra axioms);*
- *for all $s \in S$, $\eta_s : Fm_{-K} \to [0,1]$ such that $\eta_s : \varphi \mapsto V(\varphi, s)$ is a Łukasiewicz model;*
- *$V([\alpha]\varphi, s) = \bigwedge_{R_\alpha st} V(\varphi, t)$.*

*The class of all FP(KL)-models is denoted as **FP(KŁ)**. Validity is defined as expected and denoted as $\emptyset \vDash_{\textbf{FP(KŁ)}}$.*

Note that, typically, we may have $\mu_{s,i}(e) \neq \mu_{t,i}(e)$ for some s and t such that $R_\alpha st$. In other words, action α may result in a change of the subjective probability agent i assigns to event e.

We discuss a number of example scenarios involving probability dynamics and their formalizations using *FP(KŁ)*. Our main technical results are the following:

Theorem 1. *Th(**FP(KŁ)**), the set of formulas valid in all **FP(KŁ)**-models, is axiomatized by the axiom system FP(KŁ), which is an extension of the axiomatization of (crisp) modal Łukasiewicz logic [3] with the following axioms:*

(FPŁ0) $P_i(e) \to_L P_i(f)$ *if* $\vdash_{CL} e \to f$

(FPŁ1) $P_i(\top) = 1$

(FPŁ2) $P_i(\neg e) \leftrightarrow \neg P_i(e)$

(FPŁ3) $P_i(e \vee f) \leftrightarrow \neg P_i(e) \ominus P_i(e \wedge f)$

The result can be proven by the reduction inspired by in [2].

Theorem 2. *The membership problem for* $Th($***FP(KŁ)***$)$ *is decidable.*

Our decidability result is established by reduction to a version of *FP(KŁ)* with only a finite number of events, which is in turn reduced to local consequence in (crisp) modal Łukasiewicz logic, which is known to be decidable [5].

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Theory of Concepts and Intensional Interpretation of Aristotelian Logic

I will focus on the possibility of reconstructing the traditional scholastic theory of concepts as a set of marks, and then I will try to use this theory for the intensional interpretation of Aristotelian logic.

I will first try to define concepts as summaries of marks, to show what a mark of a concept is and to define what is a mark of a given concept and what is not, and to distinguish between actual, virtual and potential marks of a concept. Generally speaking, mark of a concept is a concept, which is contained in another concept. Marks of a concept are parts of the intension of a concept, which are neccesarily predicated whenever the concept is predicated. E.g., traditionally, the marks of the concept of man are the concepts of animal and rational.

I will then try to defend the theory defined in this way against Bolzano's (also traditional) objections (to be found e.g. in Materna 1998, 960-961).

Subsequently, I apply this theory to the theory of (simple) judgment, which will be understood as a question of the relationship between concepts. In the intensional interpretation of the judgments of the logical square, the judgments will be understood as follows: SaP – S contains P; SeP – S contains nonP; SiP– S does not contain nonP; SoP - S does not contain P. As this is not an extensional interpretation, quantifiers are not needed. It will be shown that in this interpretation all relations inside the logical square remain valid, simple judgments such as conversion, obversion and contraposition will also be discussed. The advantage of this approach, and its difference from many others, is that it is a purely intensional interpretation. We do not work with quantifiers here and, above all, for the same reason, the well-known and notorious problem of existential import of judgements is eliminated. For this reason, we do not need to address the familiar debates about whether existential imports have partial judgments, as the classical modern conception claims (but see Strawson 1952, 176-178), or whether existential imports have positive judgments (as Parsons claims, in Parsons 2021). Our interpretation is more along the lines of Leibniz's calculus of concepts (e.g. in Lenzen 2004), but with more or less differences, especially in the theory of negation. We work with the negation of containment and negation of concept as the analogue of the distinction between "not P" and "non-P".

Subalternation: if S contains P, then S does not contain nonP, but not vice versa (I exclude here the contradictory concepts). If S does not contain nonP, then we do not know whether S does or does not contain P. Contrariety: If S contains nonP, then S does not contain P. If S contains P, then it is not true that S contains nonP.

Subcontrariety: If S does not contain nonP, then it does not follow that S contains P, the possible option is that S does not contain P.

Obversion: I take this to mean the reciprocal determination of the relations between the underlying relations.

SaP/Se~P, SeP/Sa~P - if S contains P, then S does not contain nonP. Thus, to contain is not to be compatible with the opposite, to be incompatible is to contain the opposite. The second inference is just definitional one in my reading of SeP judgement.

SiP/So~P, SoP/Si~P - The first inference is just definitional one in my reading of SiP judgement. In the second inference, if S does not contain P, then S does not contain negation of nonP, i.e. P.

In this context, the importance and controversy of the so-called negation of the concept will be pointed out and the difference between our concept and Leibniz's concept will be shown (mainly with respect to Glashoff 2010).

In the final, I will try to show how a theory of syllogism could be interpreted. The basic concept here is a concept of distribution. A term is distributed if it is considered in its entire content. It is distributed in general judgments the predicate and in negative judgments the subject.

Although this is a basic outline, this theory can be an interesting attempt at a logic that is intensional, but not based on the modern (Carnapian) understanding of intentions as functions of possible worlds.

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Structural differences of paradoxes of self-reference

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Abstract

Priest argues that all relevant paradoxes of self-reference share a uniform structure, which he calls the *Inclosure Schema*. In contrast, we analyze the structural differences of the paradoxes of self-reference in the context of his argument and ask whether these differences provide good reasons to think that the paradoxes of self-reference do not ultimately share a uniform structure in the form of the Inclosure Schema, contra Priest.

Motivation In his paper [5], Priest makes the argument that all relevant self-referential paradoxes share a common underlying structure, which he later calls the *Inclosure Schema* (IS). Such a claim goes against the common view that there are at least two substantially different groups of paradoxes, namely semantic and set-theoretic – a division attributed mostly to Ramsey [7]. In the same paper, Priest further argues for the so-called *Principle of Uniform Solution* (PUS), which requires that, given the shared uniform structure, a proper solution to paradoxes should solve all paradoxes and not just some of them. This would also mean that all "partial" solutions (in the sense that they solve only semantic or only set-theoretic paradoxes) of paradoxes (e.g. by Tarski or Kripke, or even ZFC) are improper.

The IS is also a central concept in Priest's book *Beyond the limits of thought* [6], where, among other things, he uses the PUS as a supporting argument for the choice of dialetheism as the proper solution to paradoxes.

In the literature, one can find criticism of Priest's article mainly of two kinds. Some critics, such as in [2], [9] or [4], try to show that semantic paradoxes (like Liar paradox) do not satisfy the IS. Other critics, such as in [3], [8] or [1], on the other hand, accept (more or less) that paradoxes satisfy the IS but raise doubts about the PUS.

Inclosure Schema We use the symbols P and Q for properties, f for (possibly partial) function, X and W for sets of individuals and x for a variable of individuals (these can be sets again, but also e.g. natural numbers, propositions or sentences). The IS is not strictly formal, the concept of set and function is treated here as primitive – the justification being that we are trying to investigate paradoxes. The IS consists of the following conditions:

1. $W = \{x : P(x)\}$ exists and $Q(W)$ (Existence)
2. if X is a subset of W such that $Q(X)$:
   (a) $f(X) \notin X$ (Transcendence)
   (b) $f(X) \in W$ (Closure)

The first condition (Existence) asserts the existence of some collection W formed by the property P together with the fact that this collection W must also satisfy Q. The second and third conditions concern a function f defined in such a way that if f is applied to any subset $X \subseteq W$ satisfying Q, then its value $f(X)$ never belongs to X (Transcendence) and always belongs to W (Closure). The contradiction occurs when we apply the function f to the whole collection W (which we can do, since $Q(W)$). By the Transcendence, $f(W) \notin W$, but by the Closure, $f(W) \in W$.

The *inclosure argument* that the given paradox (represented by some triple $\langle P, Q, f\rangle$) satisfies the IS would go like this: We first give arguments that the conditions of the IS are *prima facie* satisfied and then we arrive at a contradiction.

Instances of Inclosure Schema According to Priest, Russell's paradox satisfies the IS if we choose $P =$ '$x \notin x$', $Q =$ '$X = X$' and $f(X) = X$. Then $W = \{x : x \notin x\}$ is a set of all sets that do not belong to themselves. The Transcendence condition *prima facie* holds, since if $f(X) \in X$ should hold for some $X \subseteq W$, then $X \in X$, but X should contain only those sets that do not belong to themselves. This is a contradiction. Similarly, the Closure condition *prima facie* holds since f is an identity function. The contradiction then arises at the limit case where $X = W$; then $f(W) \notin W$ and $f(W) \in W$.

The Liar paradox is also said to satisfy the IS if we choose $P =$ 'x is true', $Q =$ 'X is definable' and $f(X)$ is a sentence α where $\alpha = \langle \alpha \notin X \rangle$. Then $W = \{x : \text{'}x \text{ is true'}\}$ is a set of all true sentences. The expression $\langle \alpha \notin X \rangle$ is a sentence expressing the fact that α is not in the set X. Thus α expresses the same meaning as the sentence "This sentence is not in X.". Priest uses the property Q to ensure that the function f always returns a sentence, since it would be unclear what the function f should return if we insert an undefinable set $X \subseteq W$ into it. It is then shown that Transcendence and Closure *prima facie* hold using Tarski's T-schema. At the limit case, where $X = W$, we then get a contradiction in the form of a sentence like "This sentence is not in the set of all true sentences.".

Results Our first result is based on the realization that for most instances of the IS a contradiction occurs even outside of the limit case where $X = W$. For example, consider the paradoxical sentence "This sentence is not in the set of true sentences beginning with the letter T.", or the paradoxical set $R = \{x : \text{'}x \notin x \text{ and } x \text{ is infinite'}\}$ for which $R \in R$ and $R \notin R$ hold. By analyzing those cases where the function f is applied to proper subsets $X \subset W$ and a contradiction occurs, we have concluded that the IS makes many structural differences between the paradoxes stand out.

Our second result consists of thinking about variants of the Liar paradox that escape quite possibly any (appropriate) IS-like structure. We have in mind sentences like "This sentence is false, or satisfies the IS.".

The last result consists in discovering that the aforementioned structural differences between paradoxes can be used to formulate very simple arguments for why some instances of paradoxes *prima facie* do not satisfy Transcendence or Closure, i.e. do not satisfy the IS either.

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Philosophical Reasoning of the Alternative Set Theory

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Alternative Set Theory (AST) was created almost fifty years ago. Its first and best-known version was already published in 1979 [8]. The question remains of whether this theory is still viable and whether it is worthwhile to deal with it from other than historical perspectives [4]. From a purely mathematical point of view, AST is a special version of non-standard set theory. It is "different from the usual set theories, much weaker, but mathematically rather interesting" [5].

However, I believe that the main contribution of AST lies in the philosophical justification of its fundamental principles. Vopěnka's intention was to restore the correspondence between mathematical notions and phenomena of the natural world, to bridge the gap between infinite mathematical objects and finite physical entities which appeared after the introduction of Cantor's set theory. This endeavour has also attracted the attention of scholars from other disciplines.

Vopěnka borrowed the notion of a horizon from Husserl as the boundary that separates the field of a direct experience from that of an indirect experience. Infinite sets are sets containing *semisets*, i.e. vague parts bounded by the horizon. Continuum is described as an infinite set that forms an underlying discrete structure equipped with an indiscernibility relation. Two elements of this structure are indiscernible if their difference is beyond the horizon of our observational capacity. The concept of horizon connects the subjective point of view of an observer and the objective existence of an underlying structure.

There are several reasons why Vopěnka's phenomenological ideas have not been sufficiently appreciated. The original book [8] is mostly mathematical except for the introduction. Explanations of new ideas are contained in other works, particularly in [9], that were written in Czech and mostly have not been translated to English. Moreover, Vopěnka modified his theory several times, enriched, left blind alleys and looked for new ones. While AST partially axiomatised, the last version *New Infinitary Mathematics* [10] is open to interpretations both in a non-standard analysis and in a looser way in applied mathematics. However, the latter is more complicated and less comprehensible.

Vopěnka had always avoided directly answering the key question: "Where is the horizon?" His aim was to let this question open and to create a theory which is valid both for physical and ideal entities. But in the real world, horizon is always bound in some way, and there are many different horizons depending on the observer and the object of observation. Consequently, the relation of indiscernibility need not be transitive. If we accept a concrete number as bounding the horizon, then we encounter the same problems as the feasibility theory that is only "almost consistent". If we represent the horizon by the unbounded countable semiset of finite natural numbers, we get the same structure as the standard natural numbers in a non-standard model. [1]

I have offered a solution in [7]. To get from real-world phenomena to their mathematical representations requires always abstraction and often idealization. Abstraction can be described as a represenation that highlights some properties and disregards others, while idealization allows to change some aspects of object to obtain its ideal limit form, it is a "deliberate misrepresentation" [6].

Vopěnka describes the process of abstraction though he does not called it that way [9]. He talks about "pulling new notions out of the maze of phenomena of the natural world" . All his notions of a set, semiset, finite set, countable class, σ-class and π-class, indiscernibility relation, etc. arose as *abstractions* of phenomena of the real world. Many mathematical statements can be expressed already using these notions. Traditional mathematical concepts can be carefully replaced by these newer, more convenient ones. The novel prespective can inspire further research.

The second step is the idealization. Finite natural numbers in AST represent the *idealization* of a path toward a horizon. Real numbers which are constructed as the factorization of non-standard rational numbers modulo indiscernibility relation represent the most general idealization of continuum. Here, the indiscernibility relation is transitive and disjoint monads represent real numbers. This is a deliberate misrepresentation, the limitation of which we must be aware. The advantage is that we work in a consistent mathematical system from which we can always return to physical objects.

The themes that Vopěnka dealt with reappear recently, particularly in connection with vagueness theory, feasibility and alternatives to the classical set theory [3]; [5]; [2]. Vopěnka was no longer in contact with the international scientific community, otherwise they could have inspired each other.

In my talk, I will discuss the philosophical justification of AST and its challenge, which lies in the tension between the real and the ideal world. This implies, among other things, a natural solution of some ancient Greek paradoxes. The classical examples of continuum: space, time and motion, can be grasped in a particular way. I will mention some links to feasibility and vagueness theory and consider whether Vopěnka's phenomenological interpretation could be applied to other types of non-standard set theory.

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Strongest principles of Pure Inductive Logic

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This century has seen a revival of pure inductive logic as conceived and investigated by Rudolf Carnap and his followers (see for example [1]). As opposed to most previous investigations, the research has now been focused on polyadic languages. Rigorous mathematical foundations of the subject have been laid and considerable theory has been developed, initially summarised (along with a survey of the previous results) in the book Pure Inductive Logic [2] by Jeff Paris and the author. Subsequently, results were obtained also in the direction of further involving reasoning by analogy, adding function symbols to the framework and clarifying the roles of symmetry and irrelevance principles. In particular, since the publication of the above mentioned book, it became clear why the previously proposed 'ultimate' symmetry principle has gone too far and a suitable replacement has been found and studied, see [3]. Further evidence has accumulated in favour of the claim that one of the most powerful and best researched principles of polyadic inductive logic, Spectrum Exchangeability (Sx), is not a symmetry principle and should be motivated by considerations of irrelevance alone. A somewhat surprising result has been proved showing that Spectrum Exchangeability is in fact strictly stronger than the strongest reasonable symmetry principle, Exchangeable Invariance Principle (ENV), see [4].

In my contribution, I will outline an explanation of what Sx and ENV say, and how their relative status can be proved.

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