Cosmology of braneworlds

Basic principles & applications

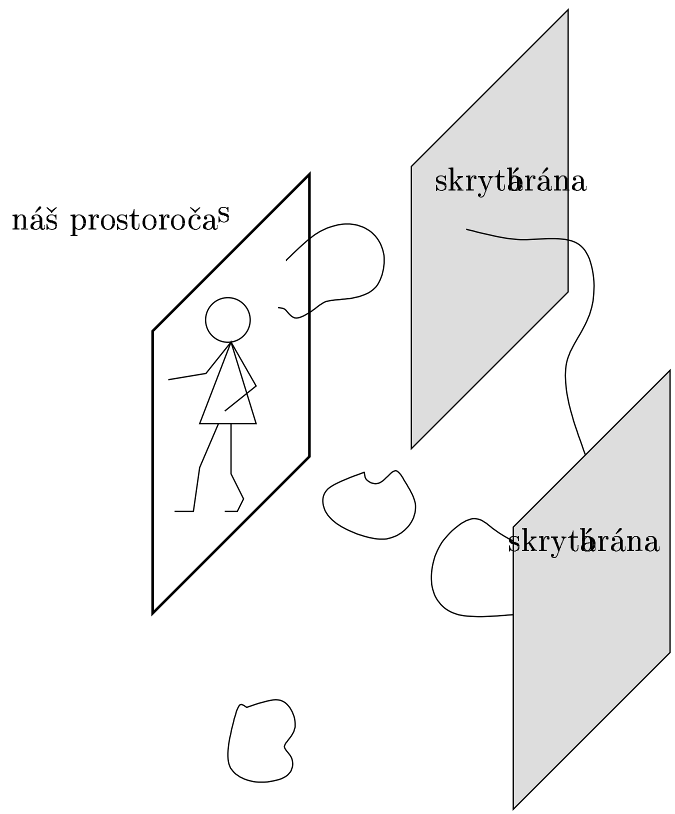
P. Klepáč

klepac@physics.muni.cz

TFA

Braneworlds

- Intuitevely: Our 4D spacetime is a timelike hypersurface in a 5D spacetime. The matter is confined to our spacetime $\mathcal{M}_4$



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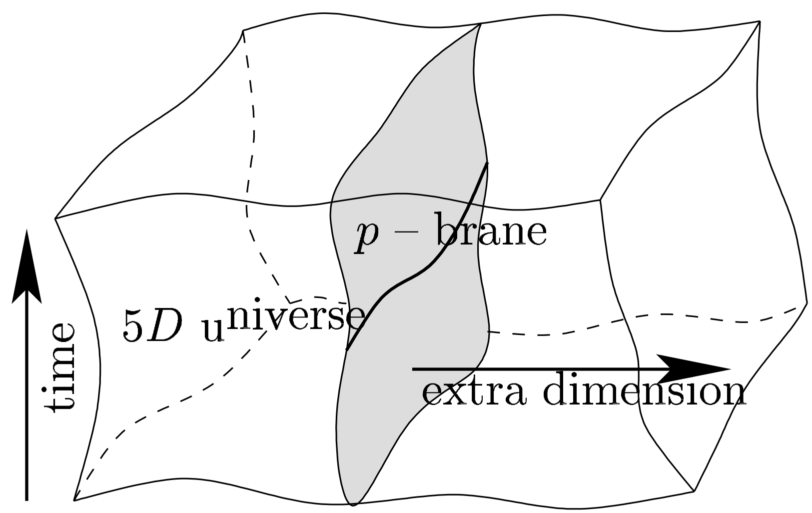

Motivation

- According to string theory, end points of open string are attached to p dim surfaces. They represent physical fields

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- Closed string can move through the bulk, their spectrun contains gravitons

Recent history

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- Randall–Sundrum (RS) braneworld – a flat braneworld embedded into 5D *AdS*

Action

- Analogical to 4D case

$$S = -\int \mathrm{d}^5x \sqrt{-g^{(5)}} \left(\frac{R}{2\kappa_5^2} + \Lambda_5 \right) + S_{\rm fields} \ ,$$

with κ_5 and Λ_5 bulk gravitational and cosmological constants

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- 5D Einstein equation

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab} R = -\kappa_5^2 T_{ab} + g_{ab} \Lambda_5$$

Major types of bulk metrics

- A static $SO(3,1)$ symmetric metric

$$\mathrm{d}s^2 = \mathrm{e}^{2A(Z)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}Z^2$$

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- An expanding braneworld

$$\mathrm{d}s^2 = a^2 b^2(\mathrm{d}t^2 - \mathrm{d}Z^2) - a^2\left(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta \mathrm{d}\phi^2\right)$$

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  - Embedding of FRW braneworld

covariant description of braneworld gravi

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- Define *jump* of a function f by

$$[f] = \lim_{\varepsilon \to +0} [f(Z+\epsilon) - f(Z-\epsilon)] \,.$$

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- Projected Einstein braneworld equations are

$$G_{ab}^{(4)} = 8\pi G \tau_{ab} - \Lambda_4 h_{ab} + \kappa_5^4 \pi_{ab} - E_{ab}$$

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$$G^{(4)}_{ab} = 8\pi G \tau_{ab} - \Lambda_4 h_{ab} + \kappa_5^4 \pi_{ab} - E_{ab} \ ,$$

where

$$\pi_{ab} = \frac{1}{12}\tau\tau_{ab} - \frac{1}{4}\tau_{ac}\tau_b{}^c + \frac{1}{8}h_{ab}\tau_{cd}\tau^{cd} - \frac{1}{24}\tau^2 h_{ab}$$

and E_{ab} is *electric Weyl tensor* $E_{ab} = C_{acbd}n^c n^d$

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  - Bianchi identity implies $\kappa_5^4 \nabla^a \pi_{ab} = \nabla^a E_{ab}$

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- Metric is

$$\mathrm{d}s^2 = \mathrm{e}^{-2k|Z|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}Z^2 \ ,$$

with $k = \sqrt{-\frac{\kappa_5^2}{6}\Lambda_5}$

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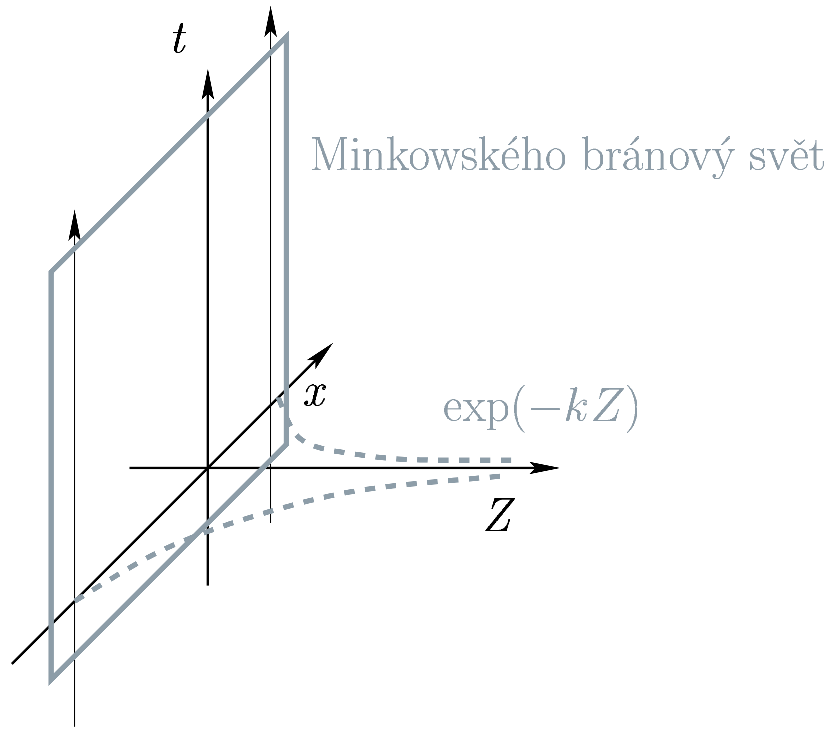

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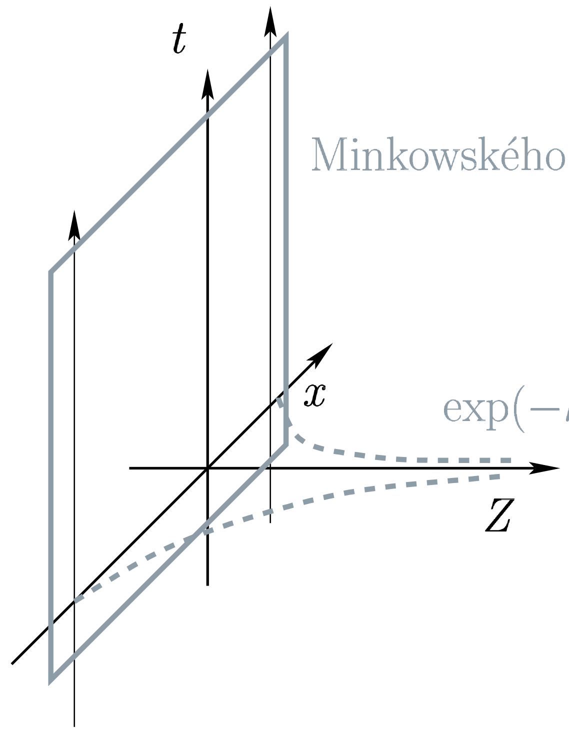

- Coordinate singularities at $Z = \pm\infty$

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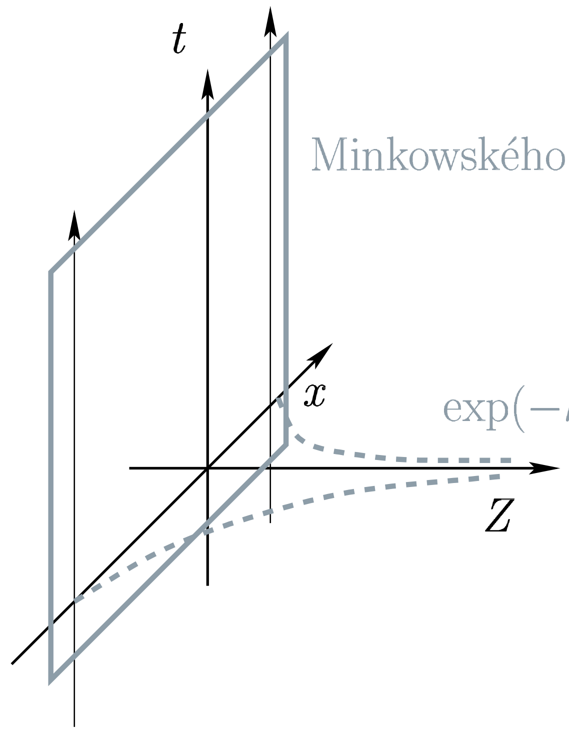
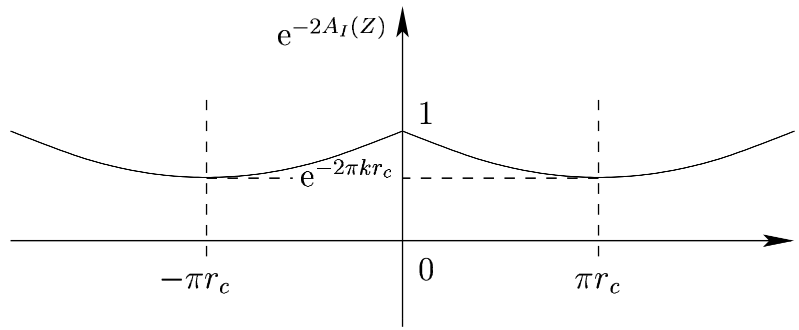


- Intrinsic singularity at $Z = 0$

RS braneworld of type I



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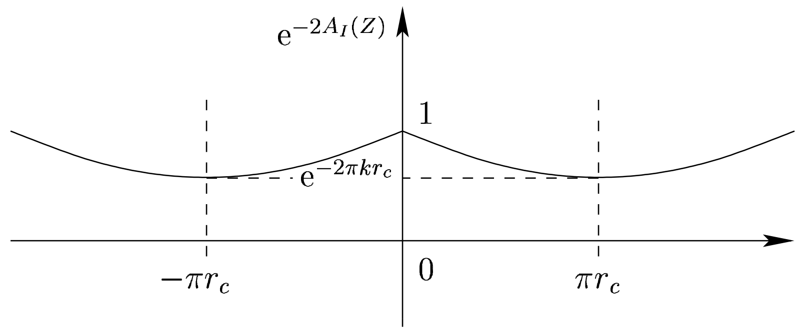

- Extra dim Z compactified

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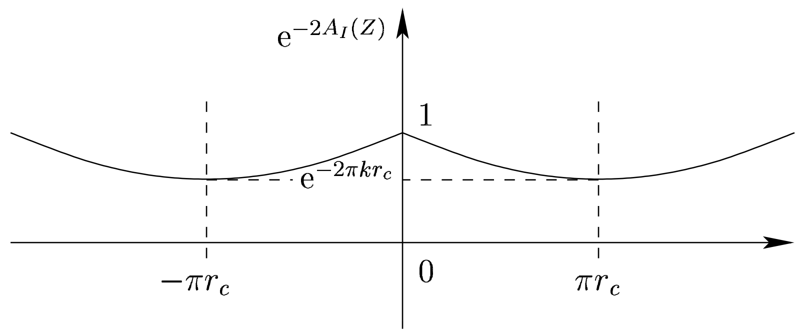

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- The physical source is constitued by two branes at $Z = 0$ and $Z = \pi r_c$

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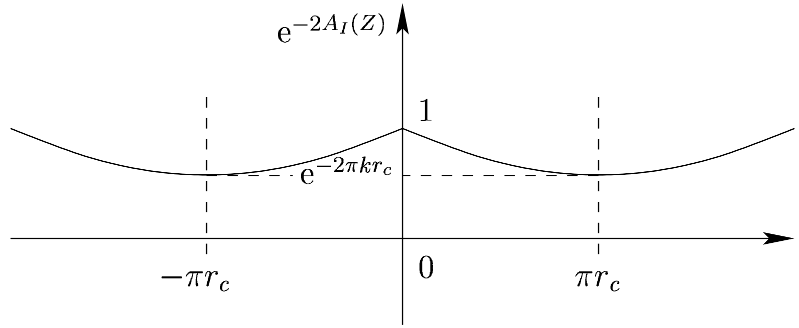

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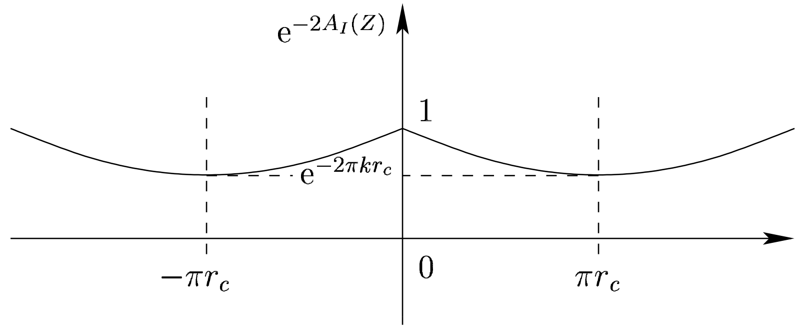

- Extra dim Z compactified
- The physical source is constitued by two branes at $Z = 0$ and $Z = \pi r_c$
- Intrinsic singularity on the branes
- Opposite brane tensions

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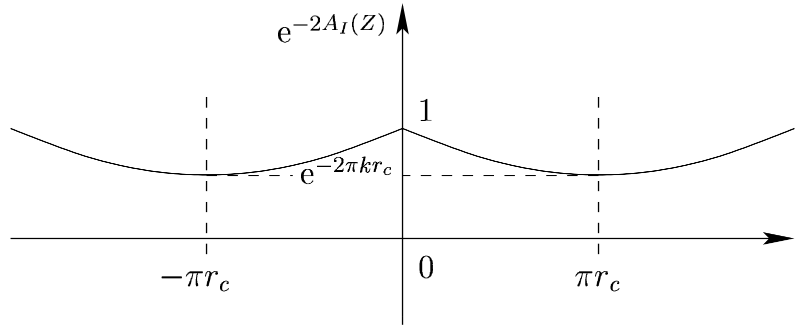

- Extra dim Z compactified
- The physical source is constitued by two branes at $Z = 0$ and $Z = \pi r_c$
- Intrinsic singularity on the branes
- Vanishing effective cosmological constants on the branes

Newtonian gravity from RS II

- Small fluctuations due to a point mass added on brane

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- Equation for $h_{\mu\nu}$ admits separation of variables, $h_{\mu\nu}(x^\rho, Z) = \psi(Z)\Phi(x^\rho)$ with

$$\partial^\mu \partial_\mu \Phi(x^\rho) = -m^2 \Phi(x^\rho)$$

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- In cylindrically symmetric case in zero–mass mode

$$\Psi(r) = -\frac{B}{r}$$

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- Kaluza–Klein non–zero modes give r^{-3} corrections

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  - Electroweak spontaneous symmetry breaking at energy scales $M_{EW} \sim 10^3 \text{ GeV}$
  - Stringy effects at scales $M_{\text{Planck}} \sim 10^{19} \text{ GeV}$
  - A huge gap between M_{Planck} and M_{EW}, 16 orders (!)

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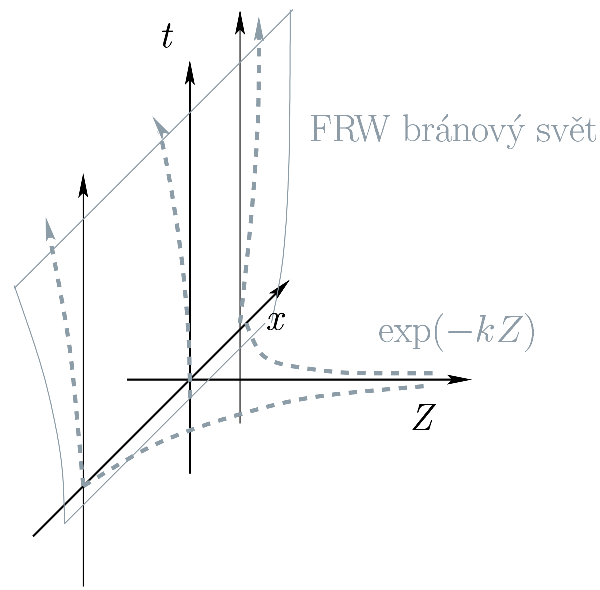
  - it is sufficient $kr_c \approx 50$ in order to obtain the hierarchy

FRW braneworld

- Embedding of FRW braneworld into 5D *AdS*



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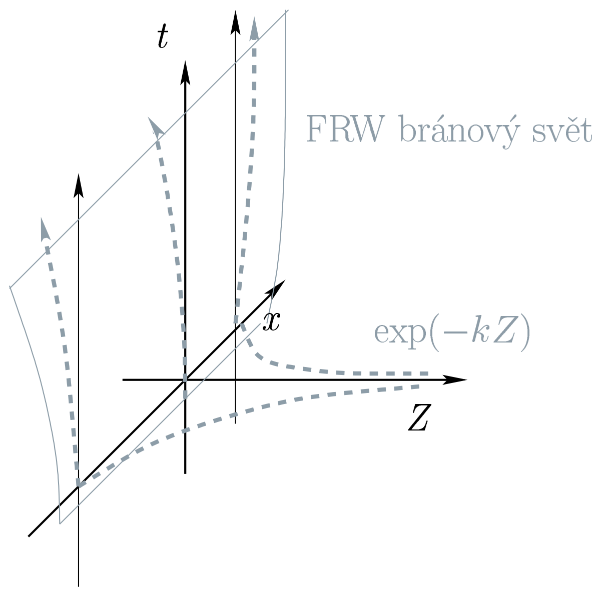

- The metric is

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FRW braneworld cosmology

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\begin{aligned}
H^2 &= \frac{8\pi G}{3}\left[\rho_M + \frac{\rho_M^2}{2\sigma}\right] + \frac{\Lambda_4}{3} + \frac{\mu}{a^4} \\
\frac{\mathrm{d}H}{\mathrm{d}\tau} &= -4\pi G(\rho_M + p_M)\left[1 + \frac{\rho_M}{\sigma}\right]
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- Quadratic term ρ_M

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- For $\rho_M >> \sigma$ it is $H \sim \rho_M$
- For $\rho_M << \sigma$ it is $H \sim \sqrt{\rho_M}$ – classical scenario

FRW braneworld cosmology

- According to classical cosmology, the FRW universe expands into “nothing”

FRW braneworld cosmology

- According to braneworld scenario, the FRW brane expands into 5D AdS spacetime