a) $\int x^3 \sin\frac{x}{2}dx$

$$u = x^3 \qquad v' = \sin\frac{x}{2}$$
$$u' = 3x^2 \qquad v = -2\cos\frac{x}{2}$$

$$= -2x^3\cos\frac{x}{2} + 6\int x^2\cos\frac{x}{2}dx =$$

$$u = x^2 \qquad v' = \cos\frac{x}{2}$$
$$u' = 2x \qquad v = 2\sin\frac{x}{2}$$

$$= -2x^3\cos\frac{x}{2} + 6\left(2x^2\sin\frac{x}{2} - 4\int x\sin\frac{x}{2}dx\right) =$$

$$= -2x^3\cos\frac{x}{2} + 12x^2\sin\frac{x}{2} - 24\int x\sin\frac{x}{2}dx =$$

$$u = x \qquad v' = \sin\frac{x}{2}$$
$$u' = 1 \qquad v = -2\cos\frac{x}{2}$$

$$= -2x^3\cos\frac{x}{2} + 12x^2\sin\frac{x}{2} - 24\left(-2x\cos\frac{x}{2} + 2\int \cos\frac{x}{2}dx\right) =$$

$$= -2x^3\cos\frac{x}{2} + 12x^2\sin\frac{x}{2} + 48x\cos\frac{x}{2} - 48\int \cos\frac{x}{2}dx =$$

$$= -2x^3\cos\frac{x}{2} + 12x^2\sin\frac{x}{2} + 48x\cos\frac{x}{2} - 96\sin\frac{x}{2} + C$$

$$\int x^3\sin\frac{x}{2}dx = -2x^3\cos\frac{x}{2} + 12x^2\sin\frac{x}{2} + 48x\cos\frac{x}{2} - 96\sin\frac{x}{2} + C$$

b) $\int e^{2x}\sin 3x dx$

$u = e^{2x}$ $\quad u' = \sin 3x$

$u' = 2e^{2x}$ $\quad u = \dfrac{-\cos 3x}{3}$

$$\int e^{2x}\sin 3x dx = \frac{-e^{2x}\cos 3x}{3} + \frac{2}{3}\int e^{2x}\cos 3x dx$$

$$3\int e^{2x}\sin 3x dx = -e^{2x}\cos 3x + 2\int e^{2x}\cos 3x dx$$

$v = e^{2x}$ $\quad u' = \cos 3x$

$v' = 2e^{2x}$ $\quad u = \dfrac{\sin 3x}{3}$

$$3\int e^{2x}\sin 3x dx = -e^{2x}\cos 3x + 2\left(\frac{e^{2x}\sin 3x}{3} - \frac{2}{3}\int e^{2x}\sin 3x\right)$$

$$3\int e^{2x}\sin 3x dx = -e^{2x}\cos 3x + \frac{2e^{2x}\sin 3x}{3} - \frac{4}{3}\int e^{2x}\sin 3x$$

$$9\int e^{2x}\sin 3x dx = -3e^{2x}\cos 3x + 2e^{2x}\sin 3x - 4\int e^{2x}\sin 3x$$

$$13\int e^{2x}\sin 3x dx = -3e^{2x}\cos 3x + 2e^{2x}\sin 3x$$

$$\int e^{2x}\sin 3x dx = \frac{2e^{2x}\sin 3x - 3e^{2x}\cos 3x}{13} + C$$

c) $\int \cos(\ln x)dx$

$\ln x = t$

$dx = e^t dt$

$\int e^t \cos t dt$

$v = e^t$ $\quad u' = \cos t$

$v' = e^t$ $\quad u = \sin t$

$$\int e^t \cos t dt = e^t \sin t - \int e^t \sin t dt$$

$v = e^t$ $\quad u' = \sin t$

$v' = e^t$ $\quad u = -\cos t$

$$\int e^t \cos t dt = e^t \sin t - \left(e^t \cos t + \int e^t \cos t dt\right)$$

$$\int e^t \cos t dt = e^t \sin t + e^t \cos t + C - \int e^t \cos t dt$$

$$2\int e^t \cos t dt = e^t(\sin t + \cos t) + C$$

$$\int e^t \cos t dt = \frac{e^t(\sin t + \cos t)}{2} + C$$

$$\int \cos(\ln x) = \frac{x}{2}(\cos(\ln x) + \sin(\ln x)) + C$$

d)

$$\int \frac{3x-4}{\left(x^2-x-6\right)^2}dx$$

Řešením $\left(x^2-x-6\right)^2$ jsou čísla x=3vx=-2

Proto platí rozklad na parciální zlomky typu:

$$\frac{A}{(x+2)^2}+\frac{B}{x+2}+\frac{C}{(x-3)^2}+\frac{D}{x-3}=$$

$$=\frac{A(x-3)^2+B(x-3)^2(x+2)+C(x+2)^2+D(x+2)^2(x-3)}{(x-3)^2(x+2)^2}=$$

$$=\frac{Ax^2-6Ax+9+(9x+2B)\left(x^2-6x+9\right)+Cx^2+4Cx+4C+\left(Dx^2+4Dx+4D\right)(x-3)}{(x-3)^2(x+2)^2}=$$

$$=\frac{x^3(B+D)+x^2(A-6B+2B+C-3D+4D)+x(-6A+9B-12B+4C-12D+4D)+(9A+18B+4C-12D)}{(x-3)^2(x+2)^2}=$$

$$\frac{x^3(B+D)+x^2(A-4B+C+D)+x(-6A-3B+4C-8D)+(9A+18B+4C-12D)}{(x-3)^2(x+2)^2}$$

Porovnáním koeficientů u týchž mocnin proměnné x v čitateli prvního a posledního zlomku dostaneme soustavu rovnic:

$$B+D=0$$
$$A-4B+C+D=0$$
$$-6A-3B+4C-8D=3$$
$$9A+18B+4C-12D=-4$$

$B=-D$:
$$A+4D+C+D=0$$
$$-6A+3D+4C-8D=3$$
$$9A-18D+4C-12D=-4$$

$$A+C+5D=0$$
$$-6A+4C-5D=3$$
$$9A+4C-30D=-4$$

$A=-C-5D$:
$$-6(-C-5D)+4C-5D=3$$
$$9(-C-5D)+4C-30D=-4$$

$$6C+30D+4C-5D=3$$
$$-9C-45D+4C-30D=-4$$

$$10C+25D=3$$
$$-5C-75D=-4$$

$$10C+25D=3$$
$$-10C-150D=-8$$

$D=\frac{1}{25}$ $C=\frac{1}{5}$ $A=-\frac{2}{5}$ $B=-\frac{1}{25}$

$$\int \frac{3x-4}{\left(x^2-x-6\right)^2}dx = -\frac{1}{5}\int \frac{dx}{(x-3)^2}+\frac{1}{25}\int \frac{dx}{x-3}+\frac{2}{5}\int \frac{1}{(x+2)^2}-\frac{1}{25}\int \frac{dx}{x+2}$$

$$\int \frac{3x-4}{\left(x^2-x-6\right)^2}dx = \frac{-1}{5(x-3)}+\frac{\ln|x-3|}{25}+\frac{2}{5(x+2)}-\frac{\ln|x+2|}{25}+C$$

e)

$$\int \frac{dx}{(x-1)\sqrt{x^2-3x+2}} = \int \frac{dx}{(x-1)\sqrt{(x-1)^2-x+1}} = \int \frac{dx}{(x-1)\sqrt{(x-1)^2-(x-1)}}$$

Substituce:

$$\left|\begin{matrix} t = x-1 \\ dt = dx \end{matrix}\right|$$

$$\int \frac{dt}{t\sqrt{t^2-t}} = \frac{2\sqrt{t^2-t}}{t}+C = \frac{2\sqrt{(x-1)^2-(x-1)}}{x-1}+C = \frac{2\sqrt{(x-1)^2\left(1-\frac{1}{x-1}\right)}}{x-1}+C = \frac{2(x-1)\sqrt{1-\frac{1}{x-1}}}{x-1}+C =$$

$$= 2\sqrt{\frac{x-2}{x-1}}+C$$

$$\int \frac{dx}{(x-1)\sqrt{x^2-3x+2}} = 2\sqrt{\frac{x-2}{x-1}}+C$$