9:

$x = a(t - \sin t) \qquad \dot{x} = a - a\cos t$

$y = a(1 - \cos t) \qquad \dot{y} = a + a\sin t$

$t \in [0, \pi]$

$$dl = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$dl = \sqrt{a^2 - 2a^2\cos t + a^2\cos^2 t + a^2\sin^2 t} = \sqrt{a^2 - 2a^2\cos t + a^2} = \sqrt{2a^2(1 - \cos t)} =$$

$$= \sqrt{2}a(1 - \cos t)$$

$$l = \int_C f dl = 2a\int_0^\pi \frac{\sqrt{1 - \cos t}}{2} dt = 2a\int_0^\pi \sin\frac{t}{2} dt = -4a\left[\cos\frac{t}{2}\right]_0^\pi = -4a(0 - 1) = 4a$$

10:

C: $x = a\cos\alpha$ $\quad \dot{x} = -a\sin\alpha$

$y = a\sin\alpha$ $\quad \dot{y} = a\cos t$

$z = bt$ $\quad \dot{z} = t$

$t \in [0;2\pi]$

$\mu = \mu_0 ... konst.$

Hmotnost: $m = \int_C \mu\, dl = \mu_0 \int_0^{2\pi} \sqrt{a^2+b^2}\, dt = \left[\mu_0 \sqrt{a^2+b^2}\, t\right]_0^{2\pi} = 2\pi\mu_0\sqrt{a^2+b^2}$

$$x_T = \frac{1}{m}\int_C \mu_0 x\, dl = \frac{1}{m}\mu_0 \int_0^{2\pi} a\cos t\sqrt{a^2+b^2}\, dt = \left[\frac{a\mu_0\sqrt{a^2+b^2}}{m}\sin t\right]_0^{2\pi} = 0$$

Těžiště:

$$y_T = ... = \frac{a\mu_0\sqrt{a^2+b^2}}{m}\int_0^{2\pi} \sin t\, dt = 0$$

$$z_T = \frac{1}{m}\mu_0 \int_0^{2\pi} bt\sqrt{a^2+b^2}\, dt = \frac{b\mu_0\sqrt{a^2+b^2}}{m}\int_0^{2\pi} t\, dt = \pi b ... = \frac{1}{2} z_{\max}$$

11:

C: $x = a\cos\alpha$ $\dot{x} = -a\sin\alpha$

$y = a\sin\alpha$ $\dot{y} = a\cos t$

$z = bt$ $\dot{z} = t$

$t \in [0;\pi]$

$\mu = \mu_0 ...konst.$

Moment setrvačnosti vůči ose y:

$$j_z = \int_C \mu_{(x;y;z)} (x^2 + y^2) dl = \int_0^\pi \overline{K}(a^2)$$

$$|\mu = \overline{K}| = \int_0^\pi \overline{K}(a^2\cos^2 t + a^2\sin^2 t)\sqrt{a^2 + b^2}dt =$$

$$= \overline{K}\sqrt{a^2 + b^2}\int_0^\pi (a^2\cos t + a^2\sin^2 t)dt = a^2\overline{K}\sqrt{a^2 + b^2}\int_0^\pi dt = a^2\mu\pi\sqrt{a^2 + b^2}$$

12:A

$F = (x, x + y)$

d... úsečka

$A = (0;0)$

$B = (b_1; b_2)$

$y = \frac{b_2}{b_1} x$

$x = t$

$y = \frac{b_2}{b_1} t$

$\dot{x} = 1$

$\dot{y} = \frac{b_2}{b_1}$

$W = \int_C \vec{F}(\vec{r}) d\vec{r}$

$W = \int_C Fxdx + Fydy = \int_{t1}^{t2} \left(Fx\left(x_{(t)}\right) \frac{dx}{dt} + Fy\left(x_{(t)}; y_{(t)}\right) \frac{dy}{dt} \right) dt$

$W = \int_0^{b1} \left(t + \left(t + \frac{b_2}{b_1} t \right) \frac{b_2}{b_1} \right) dt = \int_0^{b1} tdt + \frac{b_2}{b_1} \int_0^{b1} tdt + \left(\frac{b_2}{b_1} \right)^2 \int_0^{b1} tdt$

$W = \frac{b_1^2}{2} + \frac{b_2}{b_1} \frac{1}{2} b_1^2 + \frac{b_2^2}{b_1^2} \frac{1}{2} b_1^2 = \frac{1}{2} b_1^2 + \frac{1}{2} b_1 b_2 + \frac{1}{2} b_2^2$

12B:

$F = (x, xy)$

N...část paraboly

$y = x^2$

$A = (0;0)$

$B = (1;1)$

$x = t$

$y = t^2$

$\dot{x} = 1$

$\dot{y} = 2t$

$W = \int_C \vec{F}(\vec{r}) d\vec{r} = \int Fxdx + Fydy = \int_{t1}^{t2} \left(Fx \frac{dx}{dt} + Fy \frac{dy}{dt} \right) dt = \int_0^1 \left(t + t^3 2t \right) dt = \int_0^1 tdt + 2\int_0^1 t^4 dt = \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$