Laboratory Exercises in Astronomy — The Rotation of Mercury

DARREL B. HOFF, *University of Northern Iowa*
and GARY SCHMIDT, *Lick Observatory*

DETERMINING the planets' sizes, motions, and rotations is an important task for observational astronomy. Such information is the first step toward a deeper understanding of the solar system. Sometimes direct observations suffice, but in other cases they provide misleading answers or no answers at all. The rotation of Mercury is one case of visual observations giving entirely misleading results.

In this laboratory exercise, we will calculate Mercury's rotation period from the Doppler effect that the rotating planet produces upon radar signals reflected from it. A simple example of the Doppler effect is provided by the optical spectrum of a star moving away from or toward the observer; the wavelength (and frequency) of each line in the star's spectrum is changed by an amount proportional to the speed of approach or recession.

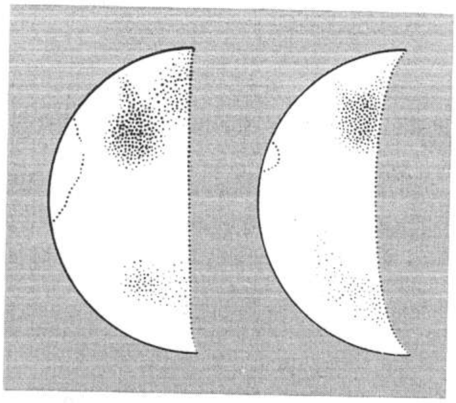
Now imagine a single line in the optical spectrum of a rotating planet. Since different parts of the planet's disk have different speeds relative to the observer, they impart different Doppler shifts, and therefore the line is broadened.

The case is closely analogous for a radar observation, where a pulse of electromagnetic energy at one frequency is beamed toward a rotating planet. The radar signal that is returned from the planet is spread out over a range of frequencies.

EARLY ATTEMPTS

Until about 1900, the only way to determine the length of a planet's day was by visually inspecting its disk for features. Mercury is particularly hard to observe because it is so close to the sun, its disk is small, and the surface features have low contrast. Using J. H. Schroeter's drawings of Mercury, F. W. Bessel deduced a rotation period of about 24 hours. It is interesting to note the supposed accuracy with which this period was known. One popular astronomy text of the mid-1800's (Elijah Burritt's *Geography of the Heavens*) gave it as 24 hours 5 minutes 28 seconds. Until the 1880's, a value near 24 hours was generally accepted.

Then, in 1889, G. V. Schiaparelli announced that he had discovered certain permanent markings on the surface of Mercury and that the planet rotated on its axis exactly once during its orbital period of 88 days. This implied that Mercury kept one face toward the sun just as the moon keeps one face toward the earth. Other visual observers, particularly Percival Lowell at Flagstaff, Arizona, appeared to confirm these findings, and the 88-day rotation period became generally accepted.



Two sketches of Mercury by E. M. Antoniadi in 1924 and 1927 with a 33-inch refractor show the visual appearance under fine conditions.

About 1900, a spectrographic method for measuring planetary rotation became available. It was first applied by J. E. Keeler to the rings of Saturn. It was known from theoretical considerations that Saturn's rings rotated as a swarm of small bodies, rather than as a rigid unit, but there was no observational proof until 1895. Keeler gave such a proof by showing that the absorption lines from the outer edge of the rings were *less* Doppler-shifted than absorption lines from the inner edge. This meant that the outer edge was revolving at a slower speed than the inner edge, just as should happen if the rings consisted of independently orbiting particles. (See "Laboratory Exercises in Astronomy: Rotation of Saturn and Its Rings," SKY AND TELESCOPE Laboratory Exercise LE03.)

The spectrographic method could be applied directly to a planet by laying the spectrograph slit across the planet's disk, parallel to its equator, and observing that lines from one edge were shifted redward, from the opposite edge toward the violet. Early this century, V. M. Slipher at Lowell Observatory and C. E. St. John and S. B. Nicholson at Mount Wilson tried this method on Mercury and Venus. They found that both planets' rotation periods were at least several days long, but could not be more specific.

A much more powerful method became feasible as a result of radar-reflection studies of the planets. Radar signals were first bounced from the moon in 1946, from Venus in 1961, and from Mercury in 1963. In August, 1965, the analysis of Doppler-broadened radar echoes finally answered the question, "What is the rotation period of Mercury?" To astronomers' surprise, it was quite different from the 88-day value in every textbook.

In this exercise, we use the same data as the original researchers. Only a millimeter scale and a hand calculator are needed.

THE RADAR OBSERVATIONS

During August, 1965, R. B. Dyce, G. H. Pettengill, and I. I. Shapiro used the 1,000-foot radio telescope at Arecibo, Puerto Rico, to beam a series of 0.0005-second and 0.0001-second radar pulses toward Mercury at a frequency of 430 megahertz. Since the round-trip travel time of the signals was much greater than the pulse length, it was possible to see how the pulses had been broadened in frequency by reflection from a rotating planet. Of course, frequency shifts can also result from motion between the planets, and from the antenna's travel around the earth's axis. Most of these effects were removed by careful timing and sampling of pulses, and by computer compensation in processing.

Supplemental Problem. The travel time itself (in seconds) can be used with the velocity of light (299,792.5 km/sec) to calculate the distance of Mercury from the earth in kilometers. Since we know the planets' distance in astronomical units at any time (from Kepler's third law), we can therefore calculate the number of kilometers in one astronomical unit. This technique is far more accurate than any classical astronomical method.

At the time of the observations used later in this exercise, Mercury's center-to-center distance from Earth was 0.617782 a.u. If the pulse two-way travel time was 616.125 seconds, how many kilometers are there in 1 a.u.?

PROCEDURE

When a radar pulse is reflected from a rotating spherical planet, the echo is spread out in time as well as frequency. The beginning of the echo is from the nearest point (sub-radar point or disk center) of the planet. After a small time delay, the echo received is from a ring-shaped area, centered on the sub-radar point. The Arecibo radar can sample the reflected pulse at various time delays.

Fig. 1 (reproduced on the next page) shows the spectrum of the radar echoes returned from Mercury for five different delay times. Note that the longer the time delay, the broader the return signal is in frequency. Successive echoes return from "rings" on Mercury farther and farther from the sub-radar point. The portion of the planet rotating toward the earth causes the return signal to have an increase in frequency (+) and the portion rotating away has a decrease (−), as in Fig. 2.

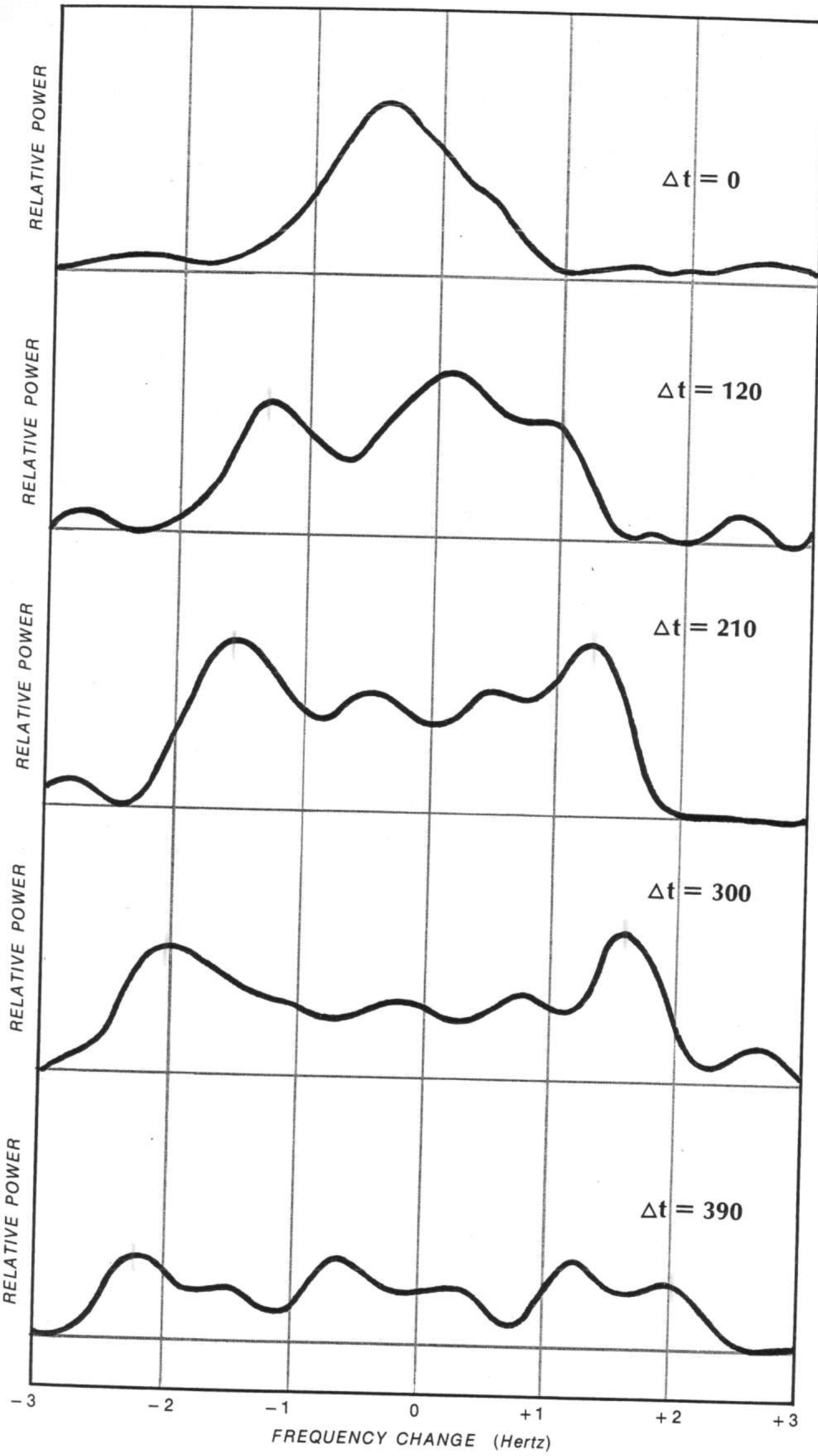


Fig. 1. The spectrum of a radar pulse returning from Mercury, sampled at five different time delays, beginning with the echo from the center of the disk (sub-radar point). The time delays (Δt) are given in microseconds (1 microsecond = 10^{-6} second). The left edge of each spectrum, showing a shift to shorter frequency, is from the portion of the planet rotating away from the observer, and the right side, showing a positive shift, is from the portion rotating toward him. This diagram has been adapted from one published by R. B. Dyce, G. H. Pettengill, and I. I. Shapiro in *Astronomical Journal*, 72, 351, 1967. It is based upon observations made on August 17, 1965, with the 1,000-foot-diameter radio telescope at Arecibo, Puerto Rico.

The increase or decrease obeys the well-known Doppler law.

In principle, it ought to be easy to determine the rotational velocity of Mercury's limb and (knowing the planet's circumference) to get the rotation period. However, the echo weakens toward the edge of the disk, and the return from the limb itself is useless. Hence, we will use the echo from a ring intermediate between the sub-radar point and the limb to obtain a line-of-sight component of Mercury's rotational velocity, and from this we will find the true rotational velocity.

To see how this is done, see Fig. 3. Recall that in Fig. 1, each signal is labeled with its time delay in microseconds. It is easy to calculate the distance d that any delayed beam has traveled beyond the sub-radar point, by multiplying half the delay time by the speed of the radar waves.

Step 1. Choose one of the time-delayed signals in Fig. 1, and for it calculate

$$d = \tfrac{1}{2} c \Delta t. \quad (1)$$

Here Δt is the time delay expressed in seconds (1 microsecond = 10^{-6} second); use 3×10^8 meters per second for c. The result is in meters.

Step 2. In Fig. 3, the lengths x and y are seen to be given by

$$x = R - d \quad (2)$$

$$y = (R^2 - x^2)^{1/2} \quad (3)$$

where R, the radius of Mercury, is 2.420×10^6 meters.

Calculate x and y. The latter will be needed in Step 4.

Step 3. Using the previously selected signal from Fig. 1, we wish to find V_0, the observed line-of-sight component of the rotational velocity at the point indicated in Fig. 3.

The Doppler equation is generally stated in terms of a change in wavelength relative to the "rest" wavelength, but it can also be stated in terms of frequency:

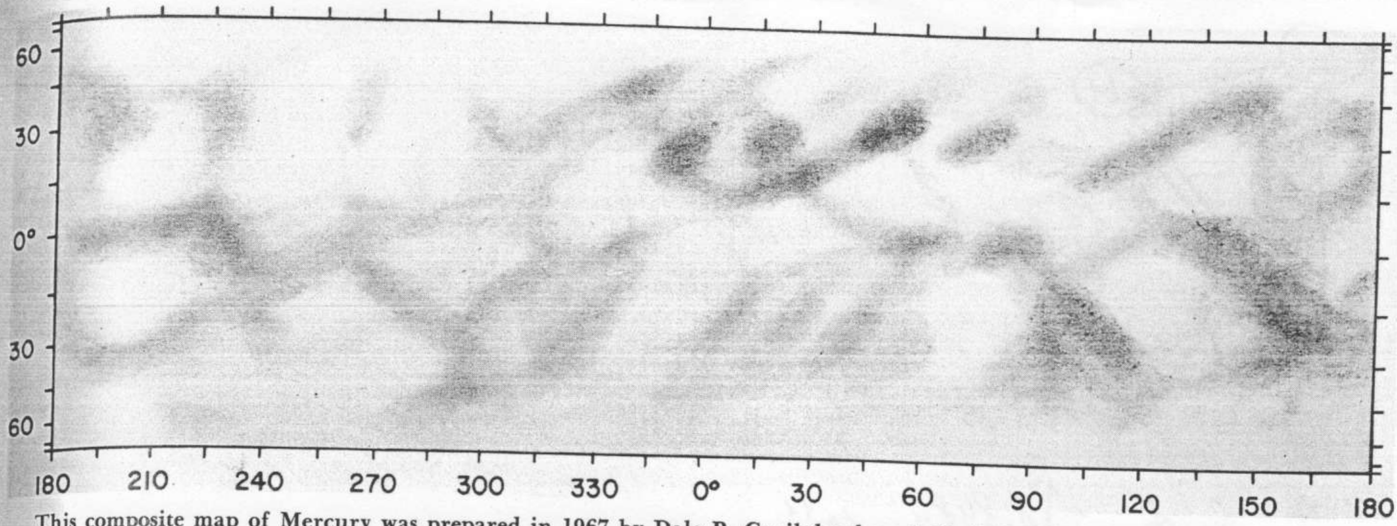
$$V_0 / c = \Delta f / f \quad (4)$$

where Δf is the change in frequency; f is the frequency of the transmitted signal (430 megahertz = 4.3×10^8 hertz); V_0 is the observed velocity; and c is the speed of the radar wave.

Examine the selected radar signal in Fig. 1, and mark with a pencil the points to the right and left where the relative power *begins* to drop down to the baseline. Read off, as accurately as you can, the frequency change at each of these points. Disregarding algebraic signs, average the results from the two shoulders. The actual Doppler frequency shift, Δf, is *half* this value, as the signal is a reflection.

Calculate V_0 in meters per second from equation (4).

Step 4. From the line-of-sight component V_0, we wish to obtain V, the unforeshortened rotational velocity. Inspection of Fig. 3 shows that the triangle containing x, y, and R is geometrically



This composite map of Mercury was prepared in 1967 by Dale P. Cruikshank and Clark R. Chapman from 130 drawings, using the correct 59-day rotation period. South is up. The longitudes here are on an arbitrary system. Since 1970, the prime meridian of Mercury has been officially defined as the meridian containing the subsolar point when Mercury was at perihelion on January 10, 1950.

similar to the triangle containing V_0 and V. Hence

$$V/V_0 = R/y. \quad (5)$$

Calculate V from (5). The result is the true rotational velocity, in meters per second.

Calculate Mercury's rotation period in seconds by dividing V into the circumference of the planet, 1.520×10^7 meters.

Finally, convert this rotation period into days (1 day = 86,400 seconds). How does your value compare with the 59 ± 3 days found by Dyce, Pettengill, and Shapiro from the total of their observations?

Note that an independent value of the rotation period can be derived from each of the profiles in Fig. 1 (apart from the one for zero delay time, of course). If you have the time, repeat Steps 1-4 for the three other signal profiles.

DISCUSSION

Later radar results have shown that the rotation period of Mercury is 58.65 days, with an uncertainty of ± 0.23 day. The Mariner 10 spacecraft flew past Mercury three times in 1974-75, obtaining high-resolution pictures from which the rotation period was very accurately determined by K. P. Klaasen as 58.6461 ± 0.005 days.

G. Colombo suspected as early as 1965 that Mercury's rotation period may be *exactly* two thirds of its 87.9693-day orbital period, or 58.6462 days. This anticipation is strongly confirmed by the excellent agreement with Klaasen's value. Mercury, therefore, exhibits a dynamical coupling of its spin rate and its orbital rate.

An interesting question is why experienced visual observers of Mercury from Schiaparelli's time until the 1960's were convinced that Mercury rotated once in 88 days, always presenting the same face to the sun. Surprisingly, about 20 detailed maps were constructed showing consistent surface markings, even though they were compiled on the basis of a wrong rotation period!

This paradox has been explained by D. P. Cruikshank and C. R. Chapman (SKY AND TELESCOPE, July, 1967, page 24). In 352 days, Mercury completes six turns on its axis and four revolutions around the sun. This is twice the interval between successive sunrises or sunsets at a fixed point on the planet. It is only a few days longer than three times Mercury's synodic period of 116 days (its cycle of phases).

Hence, after the lapse of about 350 days — which is also the interval between successive favorable apparitions — the visual observer will see the same face of the planet at the same phase. The appearances are indistinguishable from what a spurious 88-day period would produce. This situation resembles the stroboscopic effect when rotating machinery is viewed by intermittent lighting. After the true period of rotation was established as 59 days, Cruikshank and Chapman were able to combine many years of drawings and photographs into a consistent map.

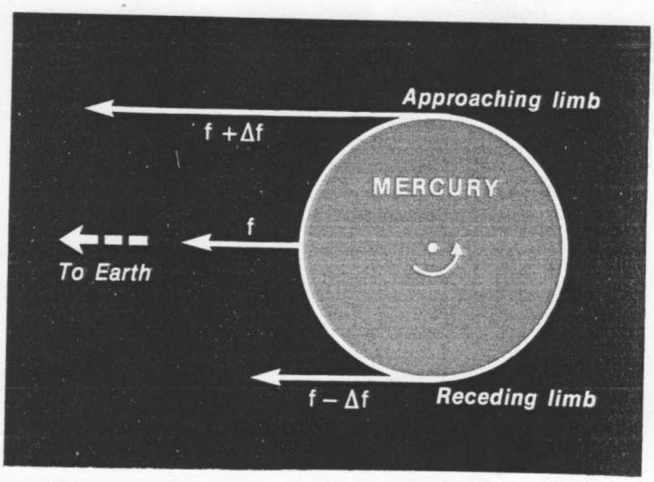


Fig. 2. A planet's rotation changes the frequency of a reflected signal as shown here. If the frequency returned from the sub-radar point is *f*, that from the approaching limb is increased to *f* + Δ*f*, while that from the receding limb is decreased to *f* – Δ*f*.

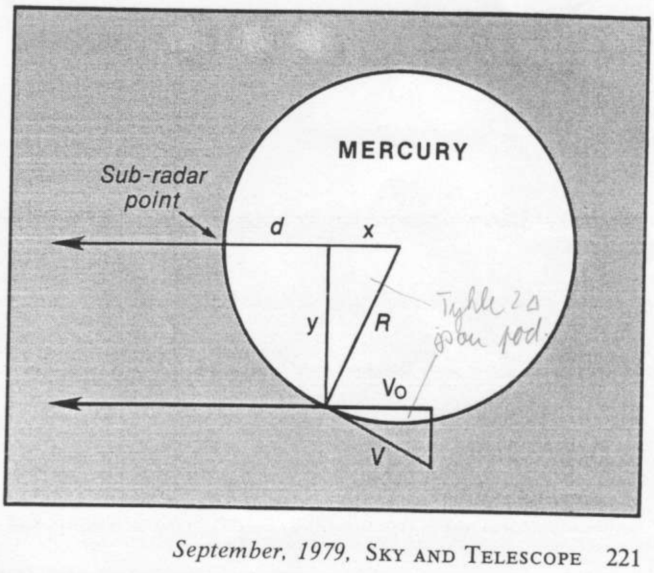


Fig. 3. Mercury's rotational speed is calculated from these geometrical relationships. R is the planet's radius; d, the delay distance; V_0, observed radial component of rotational speed at a selected point; and V, the desired full rotational speed.