

## Supplemental material description

PDF file *analysis\_of\_ellipses.pdf*:

Analysis of the ellipses in the phase space diagram for the billiard parameter  $\Phi$  close to  $\pi$ .

Video 1 (available online at

[https://www.physics.muni.cz/~tomtyc/spherical\\_wedge\\_billiard/video1.mp4](https://www.physics.muni.cz/~tomtyc/spherical_wedge_billiard/video1.mp4)):

Phase space diagram of the spherical wedge billiard for the parameter  $\Phi$  (shown at the top) ranging from 0 to  $\pi$  in steps of  $\pi/1000$ . We used 1008 initial iteration points  $(\xi, \omega)$ , with 252 equidistant values of  $\omega$  ranging from  $-\pi/2$  to  $\pi/2$  and 4 values of  $\xi$ , namely  $-\pi/2, 0, \pi/2$  and  $\pi$ . For each initial point, 1500 iterations of the Poincaré map  $M$  are shown.

Video 2 (available online at

[https://www.physics.muni.cz/~tomtyc/spherical\\_wedge\\_billiard/video2.mp4](https://www.physics.muni.cz/~tomtyc/spherical_wedge_billiard/video2.mp4)):

The disk trajectory segments with the corresponding points in the plane  $(\xi, \omega)$  for  $\Phi = 0.98 \pi$ . The initial iteration point marked by the red dot moves along the line  $\xi = \pi/2$ , the red arrow on the disk marks the corresponding initial line segment. 1500 iterations are shown.

Video 3 (available online at

[https://www.physics.muni.cz/~tomtyc/spherical\\_wedge\\_billiard/video3.mp4](https://www.physics.muni.cz/~tomtyc/spherical_wedge_billiard/video3.mp4)):

Fixed points of the mapping  $M^q$  with different orders  $q$  up to 700 in the neighborhood of the point  $\Gamma$  in the coordinates  $(\tau, \eta)$ . As the billiard parameter  $\Phi$  grows (the video shows it from  $0.745 \pi$  to  $0.751 \pi$ ), the points merge at the point  $\Gamma$ . The billiard parameter  $\Phi$  along with the angle  $\gamma$  discussed in the text is shown at the bottom.