1. Start with the (effective) Energy functional for (among other things) the Ising model

$$\int d^d x \left[\frac{1}{2}(\nabla\eta)^2 + at\eta^2 + b\eta^4 \right] \tag{1}$$

where $t = (T - T_c)/T_c$. Calulate the Free energy through a path integral using the Gaussian approximation. First find the configuration η that minimizes the action for $t \lesseqgtr 0$. If we calculate for $t > 0$ insert that value for η and expand to second order. Do the integral over all the Fourier modes of η to show that the free energy is

$$F = F_0 - \frac{1}{2} T \sum_{|k|<\Lambda} \ln \frac{2\pi VT}{2at + k^2} \tag{2}$$

Convert the sum to an integral using

$$\sum_k \to \frac{V}{(2\pi)^d} \int d^d k \tag{3}$$

Compute the heat capacity (without doing the integral). Finally do the integral and show that it diverges as $t^{-(2-\frac{d}{2})}$ for $d < 4$ but is finite for $d > 4$.

2. Find the renormalization equations and the flow diagram for the 2d Ising model in an external magnetic field on a triangular lattice. Use the method where you divide the hamiltonian into a part concerning only one blockspin and a part where blockspins interact and consider the interaction term as a perturbation.