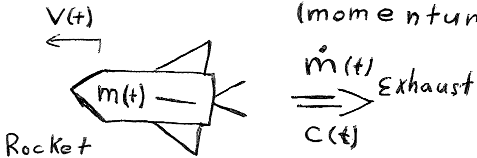
Space Propulsion AND PHYSICS OF HALL THRUSTERS

Rocket equation

(momentum balance)



$$\boxed{\frac{m_p}{m_0} = 1 - e^{-\frac{\Delta V}{c}}}$$

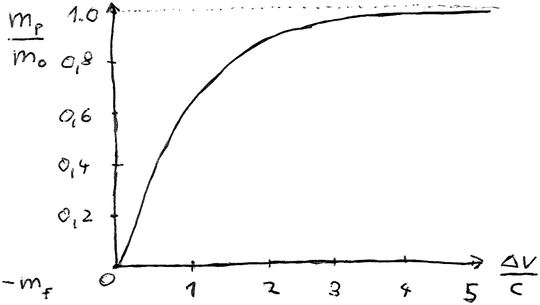
$$P_m(t) = m(t)\,V(t) + \int_0^{t'} \dot{m}(t)\,[V(t') - c(t')]\,dt'$$

$$\frac{dP_m}{dt} = 0$$

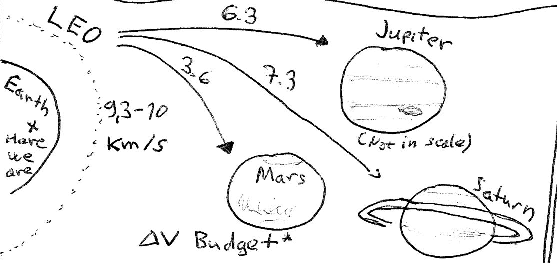
$$\dot{m} = -\frac{dm}{dt}$$

$$\int_{V_0}^{V_f} \frac{dv}{c} = \int_{m_0}^{m_f} -\frac{dm}{m}$$

$$\Delta V = V_f - V_0 \qquad m_p = m_0 - m_f$$



I. Where do you want to go?



II.

ΔV .... propellant mass consumption

Thruster firing at LEO to over-come high altitude drag.

ΔV Budget*

Dest.	Orbit radius A.U.	ΔV from LEO**
Mercury	0.39	5.5 km/s
Venus	0.72	3.5
Mars	1.52	3.6
Jupiter	5.2	6.3
Saturn	9.54	7.3
Uranus	19.19	8.0
Neptun	30.07	8.2
∞	∞	8.8

to enter orbit:
* more ΔV needed or areobraking

** without gravity assist of the moon

Specific Impulse

Force acting on the Rocket:

$$F = \dot{m} c$$

Impulse (net change of momentum:

$$I = \int_0^t F\,dt' = \int_0^t \dot{m} c\, dt' = F \cdot t \quad \begin{cases} (\text{High force} \times \text{short time}) \\ (\text{Low force} \times \text{long time}) \end{cases}$$

Objective of Propulsion systems:

Largest possible impulse to the Rocket!

$$\boxed{I_{sp} = \frac{\int_0^t \dot{m} c\,dt'}{g\int_0^t \dot{m}\,dt'} = \frac{c}{g} \quad [s]}$$

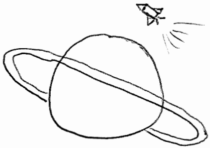
Why I_{sp} [s]?

At ground:

$$m_0 \cdot g = F = \dot{m} c = \frac{m_0 \cdot c}{t_{op}}$$

$$I_{sp} = \boxed{t_{op} = \frac{c}{g}} \quad [s]$$

Kineti~~K~~c Energy



$$K_T = \frac{1}{2} m v^2 + \int_0^t \frac{1}{2} \dot{m} (v-c)^2 \, dt'$$

Rocket — Exhaust

$$\boxed{\frac{dK_T}{dt} = \frac{1}{2} \dot{m} c^2}$$

Power goes to → acceleration of the Rocket
→ acceleration of the Exhaust

Power must come from ENERGY SOURCE.

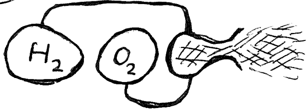
<u>Efficiency</u> of the propulsion system:

$$\boxed{\eta = \frac{\frac{1}{2} \dot{m} c^2}{P}}$$

$\frac{1}{2} \dot{m} c^2$ ← kinetic (jet) power

P ← total source power

Chemical Rockets



R: H_2	H – H	436	[kJ/mol]
R: O_2	O – O	498	
P: H_2O	H-O-H	OH: 428	
		OH–H: 498,7	

$$H_2 + \tfrac{1}{2} O_2 \rightarrow H_2O$$

Energy per unit mass: $\varepsilon = \dfrac{\varepsilon_P^{bond} - \varepsilon_R^{bond}}{m_P} = 1.34 \times 10^4 \text{ kJ/kg}$

Exhaust velocity: $c = \sqrt{2 \eta \varepsilon} = 4916 \text{ m/s} \quad (\eta = 0.9)$

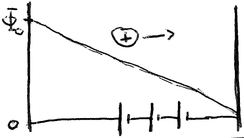
$$\boxed{I_{sp} = \frac{c}{g} = \underline{502 \text{ s}}}$$

SSME	453 s
RL-10	462 s
Merlin	348 s
Raptor	356 s

Note: "Atomic Rocket" $H + H \rightarrow H_2$ $(I_{sp} \approx 2000 \text{ s})$

Electric Rocket

H_2O^+



Energy balance for acceleration:

$$\cancel{\tfrac{1}{2} m_i u_0^2}^{\,0} + e\,\Phi_0 = \tfrac{1}{2} m_i c^2 + \cancel{e\,\Phi}^{\,0}$$

$$\Rightarrow \quad c = \sqrt{2\,\frac{e}{m_i}\,\Phi_0}\ ; \quad I_{sp} = 502\,s \quad \Rightarrow \quad \underline{\Phi_0 = 2.25\,V}$$

I_{sp} as for advanced chemical Rockets
with only <u>modest</u> applied Voltage!

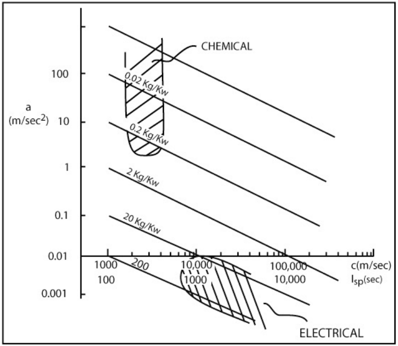
<u>Promise:</u> Larger Φ_0 breaks I_{sp} limitations of Ch. R.

<u>Downside:</u> need to carry Power source

$$\frac{\text{Power}}{\text{spacecraft mass}} = \frac{P}{m} = \frac{(\dot{m}c)\,c}{m\,2\eta} = \frac{F}{m}\,\frac{c}{2\eta} = a\,\frac{c}{2\eta}$$

‖ P/m <u>fixed</u>:
- Low acceleration
  - High specific impulse

Electric vs Chemical



Specific mass of propulsion system

$$\alpha = \frac{m_{PS}}{P}$$

Reduction of α is desirable.

Electric thrusters: kg/kW

Nuclear-Electric ~ 20

Solar-Electric ~ 200

Predicting low end:

Nuclear-Electric 1-10

Solar-Electric 80-150

Mission Analysis

Find optimal specific impulse to maximalize payload.

Wet mass: $m_0 = m_{ps} + m_p + m_s + m_{pay}$

Mass of the propulsion/power system:

$$m_{ps} = \alpha P = \frac{\alpha F c}{2\eta}$$

Mission constraints: $t_m, \Delta V, P, F, \ldots$

Optimality condition:

$$\left.\frac{d\, m_{pay}}{d c}\right|_{constraints} = 0$$

Mission Analysis (examples)

(1)

$t_m = \text{const.}$

$\dot{m} = \text{const.}$

F independent of I_{sp}

$$c_{opt} = \sqrt{\frac{2\eta\, t_m}{\alpha}}$$

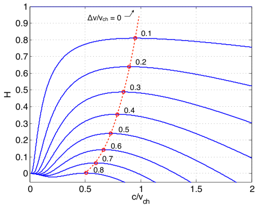
Long t_m requires large c_{opt} (I_{sp}).
(drag fighting missions at LEO)

(2) ΔV + (1) : $m_p(\Delta V)$

Objective function:

$$H = \frac{m_{pay} + m_s}{m_0}$$

$$v_{ch} = \sqrt{\frac{2\eta t_m}{\alpha}}$$ (all power to ΔV)



(3) constant voltage drop per particle acc.

$$c_{ex} \approx \sqrt{2 \frac{q}{m_i} \Delta\phi}$$

Keep $\frac{q}{m_i}$ low!

EP devices

Electro thermal: propellant is heated (resistor / arc) and expanded in a nozzle to velocity:

$$v < \sqrt{\frac{2 c_p T}{M}}$$

c_p: specific heat

T: max. nozzle temperature

Electrostatic: Ions accelerated directly by an electric field up tu velocity:

$$V_i \approx \sqrt{\frac{2 q_i U}{m_i}}$$

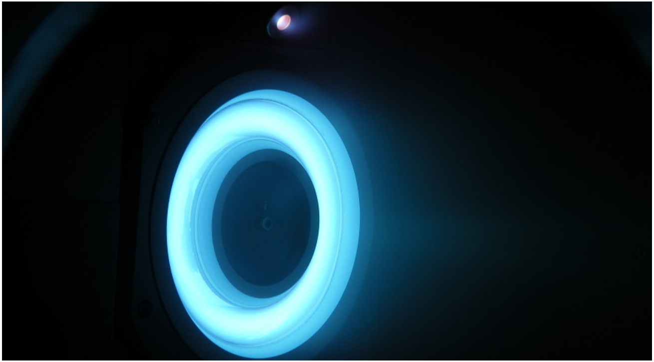
U: acceleration potential

Electromagnetic: A plasma accelerated using a combination of electric and magnetic fields.

$$\vec{F} = \vec{J} \times \vec{B}$$

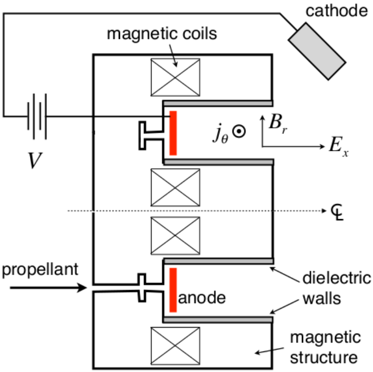
$\vec{J}$: current density

Hall thrusters



- operational status in USSR since 1980'
- many missions (>50)
- good efficiency ($\sim 50\%$) in specific impulse range 1500s
- Academic effort: ionization, electron trapping and diffusion, Lost mechanisms.

Hall thruster physics



ELECTRONS:

- electrons magnetized
- open $\vec{E} \times \vec{B}$ device
- radially confined by sheaths

Ions:

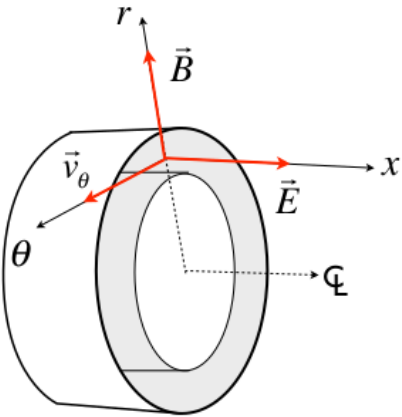
- weakly affected by $\vec{B}$
- rare collisions

$$V_i = \sqrt{\frac{ze\Phi}{m_i}}$$

(Free fall acceleration)

- Acceleration distance ~ cm (only ~1mm for ion thrusters)
- NO space charge limitations

WHY Hall thruster ?



$$\vec{V}_\theta = \frac{\vec{E} \times \vec{B}}{B^2}$$

(electron drift)

$$\vec{J}_\theta = -e\, n_e \frac{\vec{E} \times \vec{B}}{B^2}$$

(azimuthal current density)

$$\vec{F} = \vec{J}_\theta \times \vec{B}$$

(Lorentz force density)

$\Rightarrow$ Hall thruster but electrostatic accelerator

Choice of propellant

- storage requirements, spacecraft contamination, handling hazards
- impact on thruster efficiency

Assumptions:
- propellant related energy losse is mainly due to ionization
- effective ionization cost is prop. to single ε_i

$$\frac{P_{ion}}{P_{jet}} = \frac{\gamma \varepsilon_i}{\frac{1}{2} m_i c^2}$$

γ: effective ionization cost
ε_i: ionization energy
m_i: ion mass
c: specific velocity

	ε_i [eV]	m [u]	ε_i / m
→ Cs	3.9	132.9	0.029
Li	5.9	6.9	0.855
Bi	7.3	209.0	0.035
Hg	10.4	200.6	0.052
Xe	12.1	131.3	0.092
→ H	13.6	1.0	13.600
Kr	14.0	83.8	0.167
Ar	15.8	39.9	0.396

FLUID SIMULATION

$$\frac{\partial n_g}{\partial t} + \frac{\partial \left(n_g v_g\right)}{\partial z} = -n_g n K_{\mathrm{iz}} + \nu_{\mathrm{iw}} n$$

$$\frac{\partial n}{\partial t} + \frac{\partial \left(n v_{\mathrm{i}}\right)}{\partial z} = n_g n K_{\mathrm{iz}} - \nu_{\mathrm{iw}} n$$

$$\frac{\partial (n v_{\mathrm{iz}})}{\partial t} + \frac{\partial \left(n v_{\mathrm{iz}}^2 + n v_{\mathrm{B}}^2\right)}{\partial z} = n_g n K_{\mathrm{iz}} v_g + \frac{|q|}{M\mu}\left(\Gamma_0 - n v_{\mathrm{iz}}\right) - \nu_{\mathrm{iw}} n v_{\mathrm{iz}}$$

$$\frac{\partial}{\partial t}\left(\frac{T_{\mathrm{e}}^{\frac{3}{2}}}{n}\right) + \frac{\partial}{\partial z}\left(\frac{T_{\mathrm{e}}^{\frac{3}{2}} v_{\mathrm{ez}}}{n}\right) =$$

$$= \frac{\sqrt{T_{\mathrm{e}}}}{n}\left[\frac{\left(\Gamma_0 - n v_{\mathrm{iz}}\right)^2}{n^2 \mu} - n_g K_{\mathrm{iz}} \epsilon_{\mathrm{ion}} \gamma_{\mathrm{i}} - \nu_{\mathrm{ew}} \epsilon_{\mathrm{w}} - \frac{5}{2} T_{\mathrm{e}}\left(n_g K_{\mathrm{iz}} - \nu_{\mathrm{ew}}\right)\right] + \frac{T_{\mathrm{e}}^{\frac{3}{2}}}{n}\frac{\partial v_{\mathrm{ez}}}{\partial z}$$

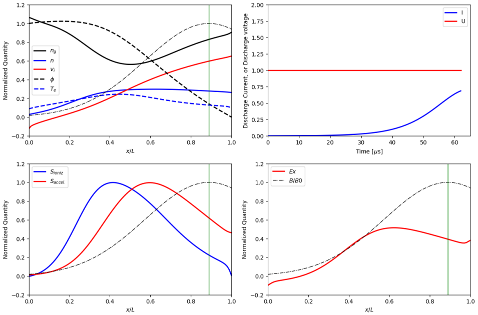


TABLE I. Simulation parameters for SPT-100.

Parameter	Value
Outer radius (R_2)	5 cm
Inner radius (R_1)	3 cm
Length of the thruster (L_0)	4 cm
Length of the simulation box (L)	5 cm
Applied voltage (V_0)	250 V
Maximum magnetic field (B_0)	200 G
Mass flow rate ($\dot{m}$)	5 mg/s
Initial gas velocity (v_{g0})	200 m/s
Initial electron temperature (T_{e0})	5 eV
Ion temperature (T_{i0})	1 eV
Ionization energy (ϵ_i)	12.1 eV
Effective ionization cost factor (γ_i)	3
Anomalous collision factor (α_B)	$\frac{1}{160}$

$$\Gamma_0 = \frac{U + \int_0^L \left[\frac{v_{\mathrm{iz}}}{\mu} + \frac{1}{n}\frac{\partial}{\partial z}(n T_{\mathrm{e}})\right] \mathrm{d}z}{\int_0^L \frac{\mathrm{d}z}{\mu n}} \qquad \Gamma_0 = n(v_{\mathrm{iz}} - v_{\mathrm{ez}}).$$

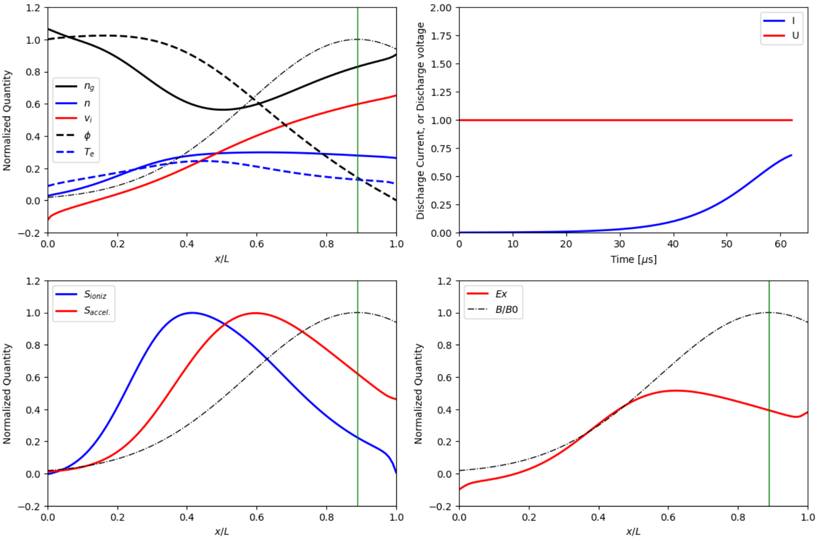
$$\mu = \frac{\frac{|q|}{\nu_{\mathrm{m}} m}}{1 + \frac{\omega_{\mathrm{c}}^2}{\nu_{\mathrm{m}}^2}} \qquad \nu_{\mathrm{iw}} = \frac{4}{3}\frac{c_{\mathrm{s}}}{R_2 - R_1} \qquad \sigma = \min\left(\frac{2T_{\mathrm{e}}}{\epsilon_{\mathrm{s}}}, 0.986\right)$$

$$\omega_{\mathrm{c}} = \frac{|q|B}{m} \qquad \nu_{\mathrm{ew}} = \frac{\nu_{\mathrm{iw}}}{1-\sigma} \qquad \nu_{\mathrm{m}} = n_g K_{\mathrm{el}} + \frac{\omega_{\mathrm{c}}}{160} + \nu_{\mathrm{ew}}$$

$$K_{\mathrm{iz}} = K_0\left(\frac{\epsilon_{\mathrm{ave}}}{\epsilon_{\mathrm{ion}}}\right)^{\frac{1}{4}} \exp\left(-2\frac{\epsilon_{\mathrm{ion}}}{\epsilon_{\mathrm{ave}}}\right) \qquad B(z) = B_{\mathrm{max}} \exp\left[-\frac{(z - l_{\mathrm{c}})^2}{l_{\mathrm{b}}^2}\right]$$

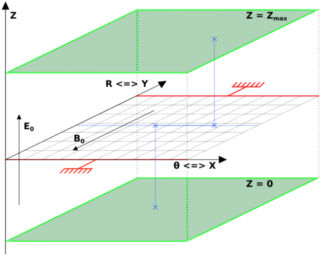
$$\epsilon_{\mathrm{w}} = 2T_{\mathrm{e}} + E_{\mathrm{kin}} + (1-\sigma)T_{\mathrm{e}} \ln\left[\sqrt{\frac{M}{2\pi m}(1-\sigma)}\right] \qquad v_{\mathrm{iz}}(0) = -\sqrt{\frac{5qT_{\mathrm{e}}(0)}{3M}}$$

$$n_g(0) = \frac{\dot{m}}{M v_g A} - \frac{n(0) v_{\mathrm{iz}}(0)}{v_g}$$

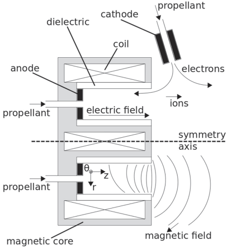


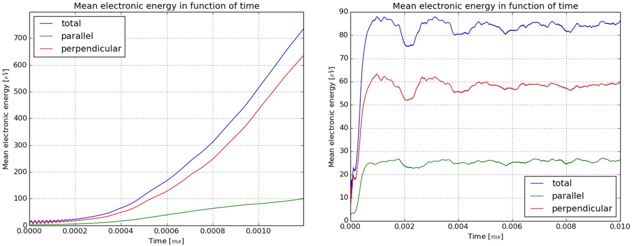
Normalized Quantity
x/L
n_g
n
v_i
ϕ
T_e
Discharge Current, or Discharge voltage
Time [μs]
I
U
S_{ioniz}
$S_{accel.}$
Ex
B/B0

PIC simulation



Parameter	Value
Gas	Xenon
L_Θ (cm)	0.5
L_R (cm)	2.0
L_z (cm)	1.0
B_0 (G)	200
E_0 (V m^{-1})	2×10^4
n_0 (m^{-3})	3×10^{17}
Δt (s)	4×10^{-12}
$\Delta x = \Delta y = \Delta z$ (cm)	2×10^{-5}
T_e (eV)	5.0
T_i (eV)	0.1
N (particles)	25×10^6
NG (gridpoints)	255×1000
N/NG (part/cell)	≈100
N_A (time-step)	2000
P_n (mTorr)	1.0
T_n (K)	300
n_g (m^{-3})	3.22×10^{19}





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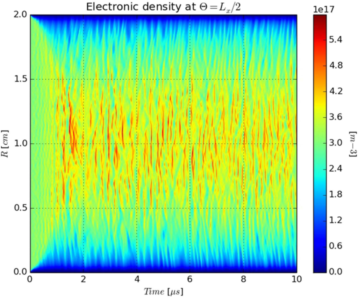
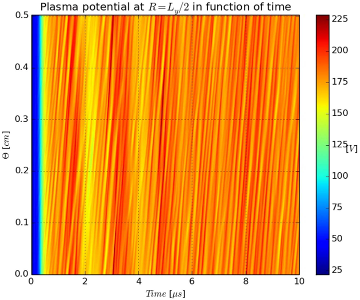
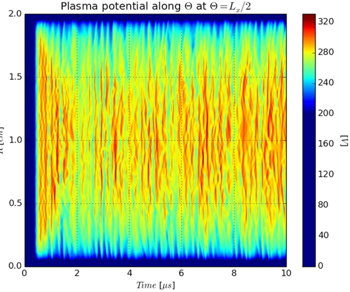
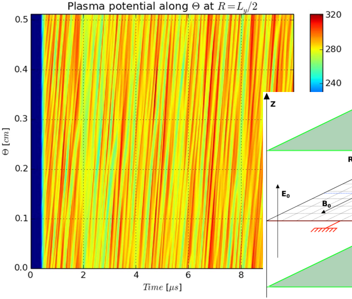
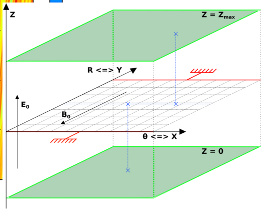
PIC simulation











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