

# Two diploma thesis proposals

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$$f(x, t) = \frac{Mvx - \frac{1}{2}Mv^2t}{\hbar}$$



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$$-i\hat{v}^\mu \partial_\mu \psi - \frac{1}{2}i\psi e^{-1} \partial_\mu (e\hat{v}^\mu) + \frac{1}{2m} e^{-1} \partial_\mu (e h^{\mu\nu} \partial_\nu \psi) - m\hat{\Phi}\psi = 0$$

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- Study Schrödinger equation in NC formulation
- Analyse the meaning of gravity in the Penrose's proposal of the collapse of the wave function.

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- Non-trivial task -since it is necessary to determine proper physical degrees of freedom.
- Important application:

## Important application

Cosmic microwave background, structure formations