Advanced Quantum Field Theory

Exam problems

1. Find the free massive propagator $\Delta(x - x')$ for a $(1+1)$-dimensional spacetime and study the behavior for $x - x'$ timelike (inside the lightcone) and $x - x'$ spacelike (outside the lightcone).

2. For a real scalar field with interaction $\lambda\varphi^4/4!$, draw all the connected Feynman diagram contributions to the two-point function $G^{(2)}$ and the four-point function $G^{(4)}$ up to order λ^3.

3. Consider a four dimensional real scalar field with the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4!}\lambda\varphi^4.$$

   Compute the lowest-order corrections to the propagator and compute the Z-factors of the quadratic terms in the Lagrangian in the $\overline{\text{MS}}$ scheme.

4. Yukawa theory is defined as a 4D theory of a Dirac fermion and real scalar field defined by the Lagrangian

$$\begin{aligned}\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}M^2\varphi^2 \\ + ig\varphi\bar{\psi}\gamma_5\psi - \frac{1}{4!}\lambda\varphi^4.\end{aligned}$$

   Derive the fermion-loop correction to the scalar propagator in the $\overline{\text{MS}}$ scheme. Show that there is an extra minus sign for a fermion loop as compared to a scalar loop.

5. Assume that starting with the Maxwell Lagrangian $\mathcal{L}_{\text{Max}}$ in 4D we fix the gauge with the 't Hooft-Veltman gauge fixing condition

$$\partial^\mu A_\mu + \frac{\lambda}{2}A^\mu A_\mu = 0,$$

   where λ is an arbitrary real constant. How would the gauge fixing term and the ghost Lagrangian look like? Show that there are now self-interactions among the photons. What is the gauge propagator? What is the ghost propagator? What are the ghost-gauge vertices? Calculate the divergent one-loop contributions to the two-point function of the gauge field. Feynman gauge $\xi = 1$ may be assumed.

6. In pure Yang-Mills theory fix the gauge using the axial gauge condition

$$n^\mu A^a_\mu = 0$$

for n^μ a fixed four-vector. Find the gluon and ghost propagators and the ghost-gluon interaction vertices.

7. In four dimensions, calculate the contribution to the Yang-Mills beta function from a non-self-interacting complex scalar field transforming in the representation R of the gauge group, coupled to a non-abelian external gauge field

$$\mathcal{L} = -D^\mu \phi^\dagger D_\mu \phi - m^2 \phi^\dagger \phi.$$

8. Consider performing the path integral for a scalar field theory in the presence of a background field $\bar{\varphi}(x)$. We define

$$e^{i\overline{W}[J,\bar{\varphi}]} = \int \mathcal{D}\varphi \; e^{iS[\varphi+\bar{\varphi}]+i\int d^dx J\varphi}.$$

Clearly $\overline{W}[J,0]$ is the original $W[J]$. We also define the quantum action in the presence of the background field

$$\overline{\Gamma}[\varphi,\bar{\varphi}] = \overline{W}[J,\bar{\varphi}] - \int d^dx \; J\varphi,$$

where now $J(x)$ is the solution of

$$\frac{\delta}{\delta J(x)}\overline{W}[J,\bar{\varphi}] = \varphi(x).$$

Show that $\overline{\Gamma}[\varphi,0]$ is equal to the original quantum action $\Gamma[\varphi]$ and that

$$\Gamma[\varphi+\bar{\varphi}] = \overline{\Gamma}[\varphi+\bar{\varphi},0] = \overline{\Gamma}[\varphi,\bar{\varphi}],$$

which means that we can calculate the original quantum action by calculating vacuum graphs in the background action

$$\Gamma[\bar{\varphi}] = \overline{\Gamma}[\bar{\varphi},0] = \overline{\Gamma}[0,\bar{\varphi}].$$

Confirm this by computing the one-loop contribution to the two-point function for a real scalar field in 4D with a $\frac{1}{4!}\varphi^4$ potential using this method and in the standard way and compare the results.