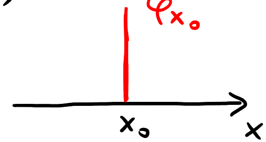
Souřadnicová a hybnostní reprezentace

- bázové funkce nepravé / zobecněné báze – nepatří do $\mathcal{F}$

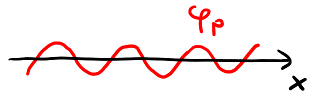
souřadnicová reprezentace

$$\varphi_{x_0}(x) = \delta(x - x_0)$$



hybnostní reprezentace

$$\varphi_p(x) = \frac{1}{\sqrt{2\pi\hbar}}\, e^{\frac{i}{\hbar}px}$$



- reprezentace stavu

$$\psi(x) = \int_{-\infty}^{\infty} dx_0\, \psi(x_0)\, \varphi_{x_0}(x) \qquad \psi(x) = \int_{-\infty}^{\infty} dp\, \tilde{\psi}(p)\, \varphi_p(x)$$

Dirac $$|\psi\rangle = \int_{-\infty}^{\infty} dx_0\, \psi(x_0)\, |x_0\rangle$$

Dirac $$|\psi\rangle = \int_{-\infty}^{\infty} dp\, \tilde{\psi}(p)\, |p\rangle$$

analogie $\psi(x) = \sum_n c_n \varphi_n(x)$ resp. $|\psi\rangle = \sum_n c_n |n\rangle$

- ortogonalita báze

$$\int_{-\infty}^{\infty} dx\ \varphi^*_{x_1}(x)\ \varphi_{x_2}(x) = \delta(x_1 - x_2)$$

$$\int_{-\infty}^{\infty} dp\ \varphi^*_{p_1}(x)\ \varphi_{p_2}(x) = \delta(p_1 - p_2)$$

Dirac $\langle x_1 | x_2 \rangle = \delta(x_1 - x_2)$

Dirac $\langle p_1 | p_2 \rangle = \delta(p_1 - p_2)$

analogie
$$\int_{-\infty}^{\infty} dx\ \varphi^*_n(x)\,\varphi_{n'}(x) = \langle n | n' \rangle = \delta_{nn'} = \begin{cases} 1 & n = n' \\ 0 & n \neq n' \end{cases}$$

- vyjádření koeficientů rozkladu (pro hybnostní rep.)

rekonstrukce

$$\tilde{\psi}(p) = \langle p | \psi \rangle = \int_{-\infty}^{\infty} dx\ \frac{e^{-\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}\ \psi(x)$$

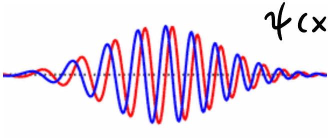
$$\psi(x) = \int_{-\infty}^{\infty} dp\ \tilde{\psi}(p)\ \frac{e^{\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}$$

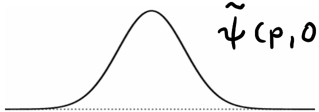
analogie
$$c_n = \langle n | \psi \rangle = \int_{-\infty}^{\infty} dx\ \varphi^*_n(x)\ \psi(x)$$

$$\psi(x) = \sum_n c_n\ \varphi_n(x)$$

Př: vlnový balík

$$\psi(x,t) = \int_{-\infty}^{\infty} dp\; \phi(p)\; e^{\frac{i}{\hbar}(px - E_p t)} \qquad \tilde{\psi}(p,t) = \sqrt{2\pi\hbar}\;\phi(p)\; e^{-\frac{i}{\hbar}E_p t}$$





$$E_p = \frac{p^2}{2m}$$

- reprezentace operátorů

$\hat{x}|\psi\rangle \;\to\; \hat{x}\,\psi(x) = x\,\psi(x)$ ($\hat{x}$ v souřadnicové je triviální)

$$\hat{x}\,\tilde{\psi}(p) = \hat{x}\int_{-\infty}^{\infty} dx\, \frac{e^{-\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}\,\psi(x) = \int_{-\infty}^{\infty} dx\; x\, \frac{e^{-\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}\,\psi(x) = \int_{-\infty}^{\infty} dx\; i\hbar\frac{d}{dp}\, \frac{e^{-\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}\,\psi(x) = \overbrace{i\hbar\frac{d}{dp}}^{\hat{x} \text{ v hybnostní}}\,\tilde{\psi}(p)$$

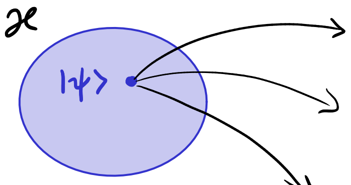
- střední hodnota

$$\langle x\rangle = \langle\psi|\hat{x}|\psi\rangle = \int_{-\infty}^{\infty} dx\; \psi^*(x)\; x\; \psi(x) = \int_{-\infty}^{\infty} dp\; \tilde{\psi}^*(p)\; i\hbar\frac{d}{dp}\;\tilde{\psi}(p)$$

souřadnicová hybnostní

Abstraktní Hilbertův prostor a reprezentace

- stavy



$|\psi\rangle$ = stav systému

souřadnicová reprezentace $\psi(x) = \langle x|\psi\rangle$

hybnostní reprezentace $\tilde{\psi}(p) = \langle p|\psi\rangle$

reprezentace založená na bázi $\{|n\rangle\}_{n=1}^{\infty}$

vektor koeficientů $c_1, c_2, c_3 \dots \qquad c_n = \langle n|\psi\rangle$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$

bázové funkce

$|x_0\rangle \leftrightarrow \delta(x-x_0)$

$|p\rangle \leftrightarrow \dfrac{e^{\frac{i}{\hbar}px}}{\sqrt{2\pi\hbar}}$

$|n\rangle \leftrightarrow \varphi_n(x)$

- operátory $\quad \hat{x} : |\psi\rangle \to \hat{x}|\psi\rangle$

$\psi(x) \to x\psi(x) \qquad \tilde{\psi}(p) \to i\hbar \dfrac{d}{dp}\tilde{\psi}(p)$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$