Perturbation theory & Feynman diagrams – part 2

① Overview

- motivation: $\hat{H} = \hat{H}_0 + \hat{V}$

$$\hat{H}_0 = \sum_{k\sigma} \varepsilon_k \hat{c}^{+}_{k\sigma} \hat{c}_{k\sigma}$$ non-interacting particles (electrons in a single band)

$$\hat{V} = \frac{1}{2} \sum_{kk'q\sigma\sigma'} V_q \hat{c}^{+}_{k+q\sigma} \hat{c}^{+}_{k'-q\sigma'} \hat{c}_{k'\sigma'} \hat{c}_{k\sigma}$$ pair interaction (Coulomb interaction)

start with $\hat{H}_0$ and include $\hat{V}$ perturbatively up to infinite order

- non-stationary perturbation theory

Dyson series

$$|\tilde{\psi}(t)\rangle = \left[\sum_{n=0}^{\infty} \left(-\frac{i}{\hbar}\right)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \cdots \int_0^{t_{n-1}} dt_n \; \tilde{V}(t_1)\, \tilde{V}(t_2) \cdots \tilde{V}(t_n) \right] |\tilde{\psi}(0)\rangle$$

interaction picture

$$|\tilde{\psi}(t)\rangle = e^{\frac{i}{\hbar}\hat{H}_0 t} |\psi(t)\rangle \qquad \tilde{V}(t) = e^{\frac{i}{\hbar}\hat{H}_0 t}\, \hat{V}(t)\, e^{-\frac{i}{\hbar}\hat{H}_0 t}$$

- thermal propagators

$$\mathcal{G}(\tau,\tau') = -\frac{1}{\hbar}\,\frac{\mathrm{Tr}\left[e^{-\beta\hat{H}}\,T\{\hat{A}(\tau)\,\hat{B}(\tau')\}\right]}{\mathrm{Tr}\left[e^{-\beta\hat{H}}\right]}$$

Heisenberg operators

$$\hat{A}(\tau) = e^{\frac{\tau}{\hbar}\hat{H}}\,\hat{A}\,e^{-\frac{\tau}{\hbar}\hat{H}}$$

interaction picture

$$\tilde{A}(\tau) = e^{\frac{\tau}{\hbar}\hat{H}_0}\,\hat{A}\,e^{-\frac{\tau}{\hbar}\hat{H}_0}$$

- Dyson expansion of the thermal propagator

$$\mathcal{G}(\tau\geq 0) = -\frac{1}{\hbar}\,\frac{\langle T\{\mathcal{U}(\hbar\beta)\,\hat{c}_A(\tau)\,\hat{c}_B^{\dagger}(0)\}\rangle_0}{\langle \mathcal{U}(\hbar\beta)\rangle_0}$$

$$\mathcal{U}(\hbar\beta) = \hat{1} + \frac{1}{1!}\left(-\frac{1}{\hbar}\right)^1 \int_0^{\hbar\beta} d\tau_1\,\tilde{V}(\tau_1) + \ldots$$

averages for a non-interacting system

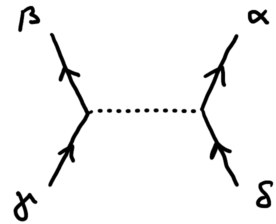
$$\mathcal{G}(\tau\geq 0) = -\frac{1}{\hbar}\,\frac{\left\langle T\left\{\sum_{n=0}^{\infty}\left(-\frac{1}{\hbar}\right)^n\frac{1}{n!}\int_0^{\hbar\beta}d\tau_1\int_0^{\hbar\beta}d\tau_2\cdots\int_0^{\hbar\beta}d\tau_n\,\tilde{V}(\tau_1)\,\tilde{V}(\tau_2)\cdots\tilde{V}(\tau_n)\;\tilde{A}(\tau)\,\tilde{B}(0)\right\}\right\rangle_0}{\left\langle T\left\{\sum_{n=0}^{\infty}\left(-\frac{1}{\hbar}\right)^n\frac{1}{n!}\int_0^{\hbar\beta}d\tau_1\int_0^{\hbar\beta}d\tau_2\cdots\int_0^{\hbar\beta}d\tau_n\,\tilde{V}(\tau_1)\,\tilde{V}(\tau_2)\cdots\tilde{V}(\tau_n)\right\}\right\rangle_0}$$

$\tilde{V}, \tilde{A}, \tilde{B}$ expressed using c^{+}, c $\xrightarrow{\text{Wick's theorem}}$ products of non-interacting $\mathcal{G}_0$

- Feynman diagrams

$$\tilde{V}(\tau) = \frac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\ \hat{c}^{+}_{\alpha}(\tau^{+})\, \hat{c}^{+}_{\beta}(\tau^{+})\, \hat{c}_{\gamma}(\tau)\, \hat{c}_{\delta}(\tau)$$

labels of single-particle states



useful to keep track of the terms in the perturbation series

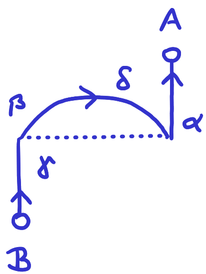
$$\mathcal{G} \sim -\frac{1}{\hbar}\left\langle T\left\{ \hat{c}_{A}(\tau)\, \hat{c}^{+}_{B}(0) \right\}\right\rangle$$ ← Heisenberg

interaction picture

$$\mathcal{G} \sim \frac{1}{\cdots}\left[\ldots + \left\langle T\left\{ \hat{c}_{A}(\tau)\, \hat{c}^{+}_{\alpha}(\tau_1^{+})\, \hat{c}^{+}_{\beta}(\tau_1^{+})\, \hat{c}_{\gamma}(\tau_1)\, \hat{c}_{\delta}(\tau_1)\, \hat{c}^{+}_{B}(0) \right\}\right\rangle_0 + \ldots \right]$$

Hartree-Fock terms

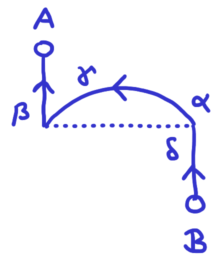
$c_A\, c^{+}_{\alpha}\, c^{+}_{\beta}\, c_{\gamma}\, c_{\delta}\, c^{+}_{B}$



τ

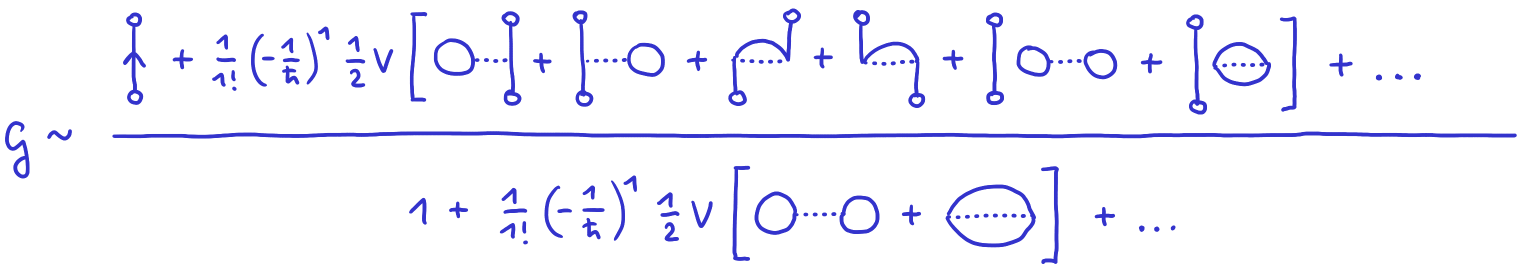
τ_1

$\tau' = 0$



$c_A\, c^{+}_{\alpha}\, c^{+}_{\beta}\, c_{\gamma}\, c_{\delta}\, c^{+}_{B}$

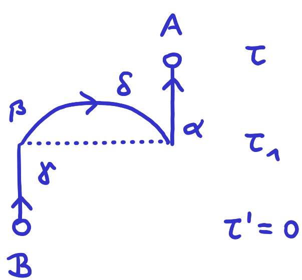
GOAL 1: reduction of the series (combinatorial explosion of terms with increasing n)



GOAL 2: diagrammatic rules – how to convert a diagram into an expression

$n=1$ in the numerator : $\langle T\{\hat{c}_A(\tau)\,\hat{c}^+_\alpha(\tau_1^+)\,\hat{c}^+_\beta(\tau_1^+)\,\hat{c}_\gamma(\tau_1)\,\hat{c}_\delta(\tau_1)\,\hat{c}^+_B(0)\}\rangle_0$

$c_A\, c^+_\alpha\, c^+_\beta\, c_\gamma\, c_\delta\, c^+_B$

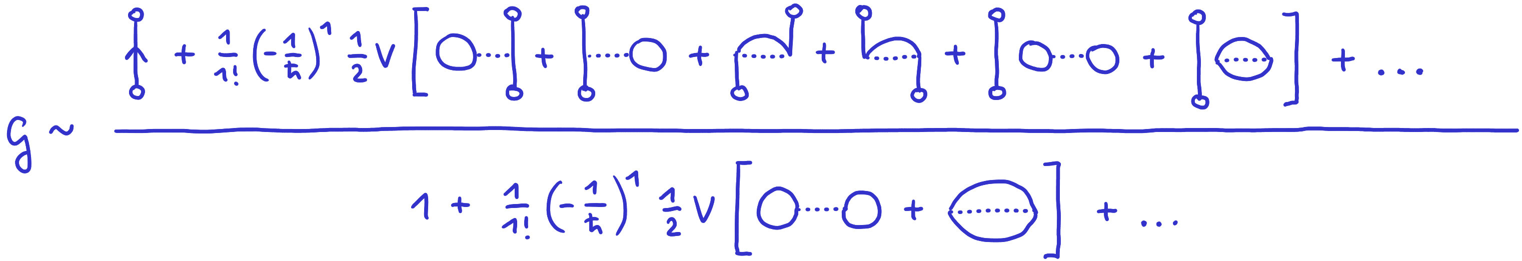


$$-\frac{1}{\hbar}\quad\left(-\frac{1}{\hbar}\right)^1\frac{1}{1!}\quad\int_0^{\hbar\beta}d\tau_1\quad\sum_{\alpha\beta\gamma\delta}\quad\frac{1}{2}V_{\alpha\beta\gamma\delta}$$

$$\pm\quad \mathcal{G}_0(A,\alpha,\tau-\tau_1)\quad \mathcal{G}_0(\delta,\beta,0^-)$$

$$\mathcal{G}_0(\gamma,B,\tau_1-\tau')$$

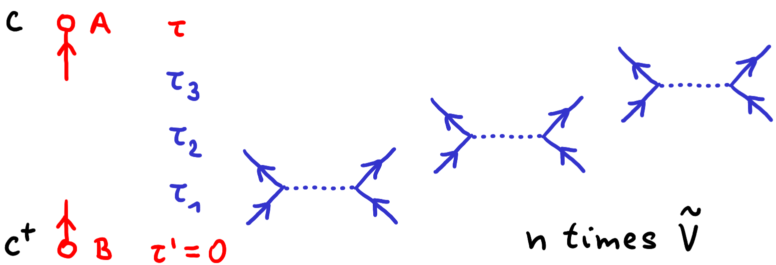
② Reduction of the number of diagrams ($\infty \rightarrow$ smaller ∞)



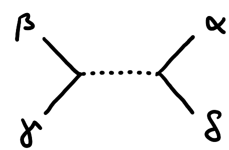
- cancellation of disconnected diagrams

numerator of single-particle propagator

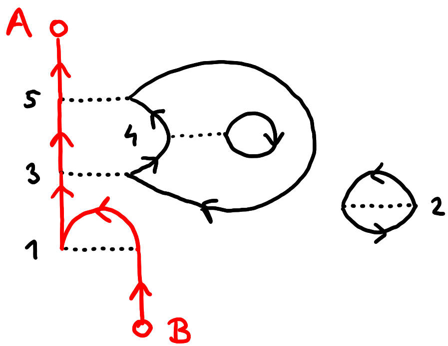
$$\left\langle T\left\{ \sum_{n=0}^{\infty} \left(-\frac{1}{\hbar}\right)^n \frac{1}{n!} \int_0^{\hbar\beta} d\tau_1 \int_0^{\hbar\beta} d\tau_2 \cdots \int_0^{\hbar\beta} d\tau_n \, \tilde{V}(\tau_1)\, \tilde{V}(\tau_2) \cdots \tilde{V}(\tau_n)\, \hat{c}_A(\tau)\, \hat{c}_B^{+}(0) \right\}\right\rangle_0$$

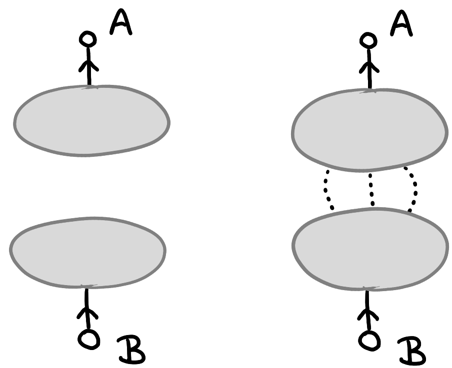


$$\tilde{V} = \sum V c_\alpha^+ c_\beta^+ c_\gamma c_\delta$$



A and B always connected by a backbone of the diagram – continuous ↑ line

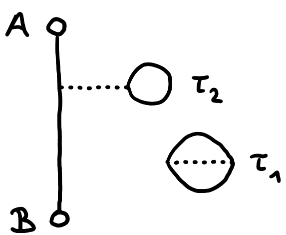




odd number of c, c^+

odd number of c, c^+

diagram of order n → n interaction lines 



r interaction lines connected to the backbone $0 \le r \le n$

$n-r$ interaction lines in disconnected parts

$$\underbrace{\int \text{over } \tau_i \text{ set connected to } A, B}_{\substack{r \text{ integrals,} \\ \tau, \tau' \text{ remaining variable}}} \times \underbrace{\int \text{over } \tau_i \text{ set disconnected from } A, B}_{\substack{n-r \text{ integrals} \\ \text{(can factorize further)}}}$$

implies sums over corresponding $\alpha, \beta, \gamma \ldots$

distribution into r and $n-r$ lines can be done in $\binom{n}{r}$ ways

$$\mathcal{G}\ \text{numerator} = \sum_{n=0}^{\infty} \frac{1}{n!}\Big(\underbrace{-\frac{1}{2\hbar}}_{\text{from pair } \tilde{V}}\Big)^n \Big[\text{all diagrams with } n \cdots\cdots \text{ lines}\Big]$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\Big(-\frac{1}{2\hbar}\Big)^n \sum_{r=0}^{n} \underbrace{\binom{n}{n-r}}_{\frac{n!}{(n-r)!\,r!}} \underbrace{\left[\begin{array}{c}\text{diagrams with } r \cdots\cdots \\ \text{lines connected to AB}\end{array}\right]}_{\substack{r \text{ integrals over } \tau_i \\ \text{and corresponding } \sum_{\alpha\ldots}}} \underbrace{\left[\begin{array}{c}\text{disconnected diagrams} \\ \text{containing } n-r \cdots\cdots \text{ lines}\end{array}\right]}_{n-r \text{ integrals over } \tau_i}$$

$$\sum_{n=0}^{\infty} \sum_{r=0}^{n} \frac{1}{r!}\Big(-\frac{1}{2\hbar}\Big)^r \left[\begin{array}{c}\text{diagrams with } r \cdots\cdots \\ \text{lines connected to AB}\end{array}\right] \frac{1}{(n-r)!}\Big(-\frac{1}{2\hbar}\Big)^{n-r} \left[\begin{array}{c}\text{disconnected diagrams} \\ \text{containing } n-r \cdots\cdots \text{ lines}\end{array}\right]$$

$$\sum_{m=0}^{\infty} \frac{1}{m!}\Big(-\frac{1}{2\hbar}\Big)^m \left[\begin{array}{c}\text{diagrams with } m \cdots\cdots \\ \text{lines connected to AB}\end{array}\right] \underbrace{\sum_{m'=0}^{\infty} \frac{1}{m'!}\Big(-\frac{1}{2\hbar}\Big)^{m'} \left[\begin{array}{c}\text{disconnected diagrams} \\ \text{containing } m' \cdots\cdots \text{ lines}\end{array}\right]}_{\text{denominator}}$$

$$\longrightarrow \quad G(\tau \geq 0) = -\frac{1}{\hbar} \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{1}{2\hbar}\right)^n \left[\begin{array}{l} \text{diagrams with } n \text{ ......} \\ \text{lines connected to } AB \end{array} \right]$$

- cancellation of $\frac{1}{n!}$

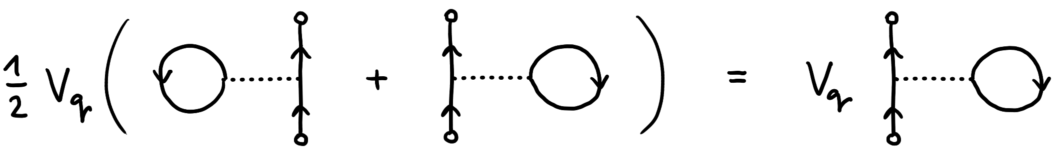
permuting $\tau_1 \dots \tau_n$ only changes the vertical stacking of time labels

$\rightarrow$ $n!$ equivalent contributions $\rightarrow$ $\frac{1}{n!}$ cancelled

Example:

- cancellation of $\frac{1}{2}$

each interaction line can be used in two orientations $\longrightarrow$ compensation of $\frac{1}{2^n}$

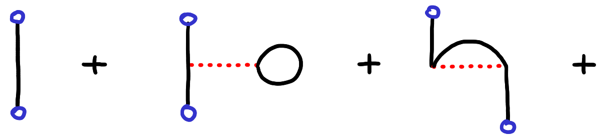


$$\longrightarrow \quad \mathcal{G}(\tau \geq 0) = -\frac{1}{\hbar} \sum_{n=0}^{\infty} \left(-\frac{1}{\hbar}\right)^n \left[\begin{array}{l} \text{topologically inequivalent diagrams with} \\ n \;\cdots\cdots\; \text{interaction lines connected to } A, B \end{array} \right]$$

up to second order:

$n = 0$ $\qquad\qquad$ $n = 1$

$\mathcal{G}(\tau \geq 0) =$



③ Conservation of momentum (+ spin)

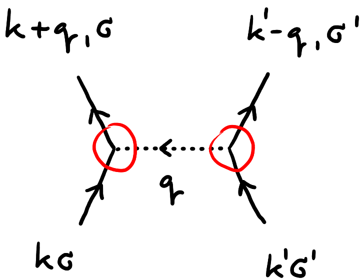
- translationally invariant system
  $\longrightarrow$ momentum conservation
- spin isotropic system
  $\longrightarrow$ spin conservation

single-particle propagator

$$\mathcal{G}_0(k,\tau) = -\frac{1}{\hbar}\left\langle T\left\{ \hat{c}_{k\sigma}(\tau)\, \hat{c}^{\dagger}_{k\sigma}(0) \right\}\right\rangle_0$$

(the same for $\mathcal{G}$)

- conserving interaction – Coulomb interaction



$$V = \frac{1}{2}\sum_{kk'q\sigma\sigma'} V_q\, \hat{c}^{\dagger}_{k+q\,\sigma}\, \hat{c}^{\dagger}_{k'-q\,\sigma'}\, \hat{c}_{k'\sigma'}\, \hat{c}_{k\sigma} \qquad V_q = \frac{1}{\Omega}\frac{e^2}{\varepsilon_0 q^2}$$

conservation – momentum: $k + k' = (k+q) + (k'-q)$

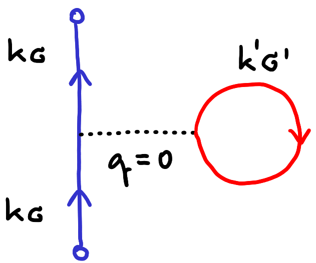
spin: $\sigma \to \sigma \qquad \sigma' \to \sigma'$

internal momentum transfer q associated with ....

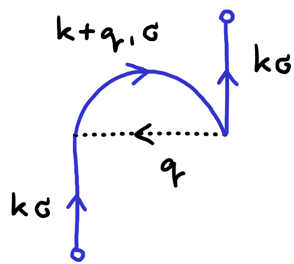
no transfer of spin through the Coulomb line

- rules for the momentum and spin "flow" through the diagram:
  1) propagators keep their momentum/spin
  2) momentum and spin conserved at each vertex

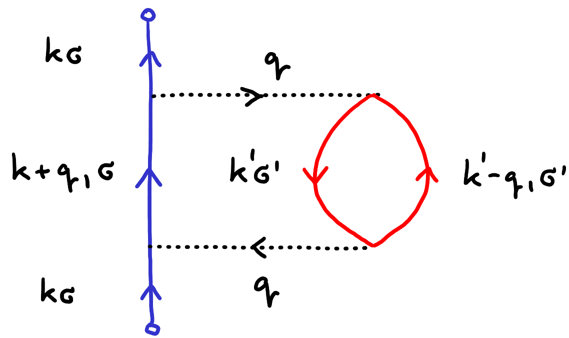
$$\mathcal{G} \sim \langle T\{ \hat{c}_{k\sigma}(\tau)\, \hat{c}^+_{k\sigma}(\tau') \}\rangle \qquad V = \frac{1}{2} \sum_{kk'q\sigma\sigma'} V_q\, \hat{c}^+_{k+q\sigma}\, \hat{c}^+_{k'-q\sigma'}\, \hat{c}_{k'\sigma'}\, \hat{c}_{k\sigma}$$



$$\sum_{k'\sigma'} V_{q=0}\, \mathcal{G}_{k\sigma}\, \mathcal{G}_{k\sigma}\, \mathcal{G}_{k'\sigma'}$$



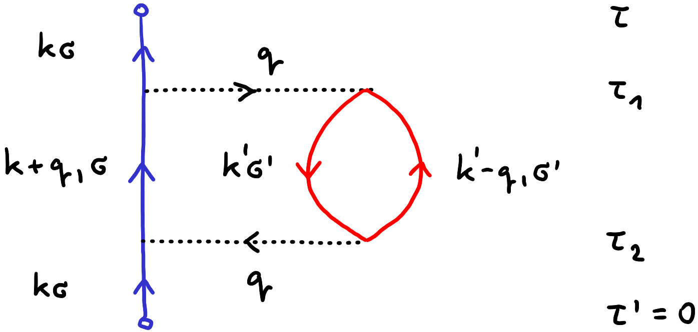
$$\sum_{q} V_q\, \mathcal{G}_{k\sigma}\, \mathcal{G}_{k+q\sigma}\, \mathcal{G}_{k\sigma}$$



$$\sum_{qk'\sigma'} V_q^2\, \mathcal{G}_{k\sigma}\, \mathcal{G}_{k+q\sigma}\, \mathcal{G}_{k\sigma}\, \mathcal{G}_{k'\sigma'}\, \mathcal{G}_{k'-q\sigma'}$$

- Free momenta and spins to be summed over

4 Conversion to Matsubara representation



$-\hbar\,\mathcal{G}(k,\tau)$ contains

$$\left(-\frac{1}{\hbar}\right)^2 \int_0^{\hbar\beta} d\tau_1 \int_0^{\hbar\beta} d\tau_2 \sum_{qk'\sigma'} \langle \cdots \rangle_0$$

$$\langle \cdots \rangle_0 = \left\langle T\left\{ c_{k\sigma}(\tau) \left(V_q c^\dagger_{k'\sigma'} c^\dagger_{k\sigma} c_{k+q\sigma} c_{k'-q\sigma'}\right)_{\tau_1} \left(V_{-q} c^\dagger_{k'-q\sigma'} c^\dagger_{k+q\sigma} c_{k\sigma} c_{k'\sigma'}\right)_{\tau_2} c^\dagger_{k\sigma}(0) \right\}\right\rangle$$

$$= V^2 \left\langle T\{c_{k\sigma}(\tau)\, c^\dagger_{k\sigma}(\tau_1)\}\right\rangle_0 \left\langle T\{c_{k+q\sigma}(\tau_1)\, c^\dagger_{k+q\sigma}(\tau_2)\}\right\rangle_0 \left\langle T\{c_{k\sigma}(\tau_2)\, c^\dagger_{k\sigma}(0)\}\right\rangle_0 \times$$

$$\times \left\langle T\{c^\dagger_{k'\sigma'}(\tau_1)\, c_{k'\sigma'}(\tau_2)\}\right\rangle_0 \left\langle T\{c_{k'-q\sigma'}(\tau_1)\, c^\dagger_{k'-q\sigma'}(\tau_2)\}\right\rangle_0$$

$$= V^2 \left[-\hbar\,\mathcal{G}_0(k,\tau-\tau_1)\right] \left[-\hbar\,\mathcal{G}_0(k+q,\tau_1-\tau_2)\right] \left[-\hbar\,\mathcal{G}_0(k,\tau_2)\right] \left[-\hbar\,(-)\,\mathcal{G}_0(k',\tau_2-\tau_1)\right] \left[-\hbar\,\mathcal{G}_0(k'-q,\tau_1-\tau_2)\right]$$

$-\hbar\mathcal{G}(k,\tau)$ contains

$$\left(-\frac{1}{\hbar}\right)^2 \int_0^{\hbar\beta} d\tau_1 \int_0^{\hbar\beta} d\tau_2 \sum_{qk'\sigma'} V_q^2 \left[-\hbar\mathcal{G}_0(k,\tau-\tau_1)\right]\left[-\hbar\mathcal{G}_0(k+q,\tau_1-\tau_2)\right]\left[-\hbar\mathcal{G}_0(k,\tau_2)\right]\left[-\hbar(-)\mathcal{G}_0(k',\tau_2-\tau_1)\right]\left[-\hbar\mathcal{G}_0(k'-q,\tau_1-\tau_2)\right]$$

- Matsubara coefficients $\quad -\mathcal{G}(k,iE) = \frac{1}{\hbar}\int_0^{\hbar\beta} d\tau \left[-\hbar\mathcal{G}(k,\tau)\right] e^{iE_n\frac{\tau}{\hbar}}$

Matsubara representation $\quad \mathcal{G}_0(k,\tau) = \frac{1}{\hbar\beta}\sum_{iE_n}\mathcal{G}_0(k,iE_n)\, e^{-iE_n\frac{\tau}{\hbar}} \qquad iE_n = i\,\frac{\pi(2n+1)}{\beta}$

$$-\mathcal{G}(k,iE) = (-1)\sum_{qk'\sigma'}\frac{1}{\hbar}\int_0^{\hbar\beta}d\tau \left(-\frac{1}{\hbar}V_q\right)\int_0^{\hbar\beta}d\tau_1 \left(-\frac{1}{\hbar}V_q\right)\int_0^{\hbar\beta}d\tau_2 \;\frac{1}{\beta^5}\sum_{iE_1}\sum_{iE_2}\sum_{iE_3}\sum_{iE_4}\sum_{iE_5}$$

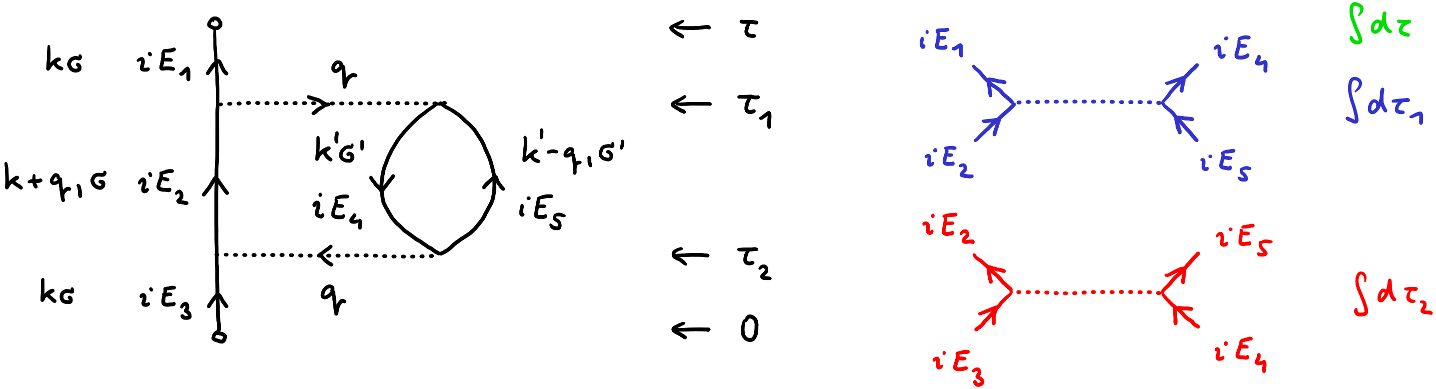
$$\left[-\mathcal{G}_0(k,iE_1)\,e^{-iE_1\frac{\tau-\tau_1}{\hbar}}\right]\left[-\mathcal{G}_0(k+q,iE_2)\,e^{-iE_2\frac{\tau_1-\tau_2}{\hbar}}\right]\left[-\mathcal{G}_0(k,iE_3)\,e^{-iE_3\frac{\tau_2}{\hbar}}\right]$$

$$\left[-\mathcal{G}_0(k',iE_4)\,e^{-iE_4\frac{\tau_2-\tau_1}{\hbar}}\right]\left[-\mathcal{G}_0(k'-q,iE_5)\,e^{-iE_5\frac{\tau_1-\tau_2}{\hbar}}\right] e^{iE_n\frac{\tau}{\hbar}}$$

- τ-integrals imply "energy conservation" : $\frac{1}{\hbar}\int_0^{\hbar\beta}d\tau\, e^{iE\frac{\tau}{\hbar}}\, e^{-iE'\frac{\tau}{\hbar}} = \beta\,\delta_{iE,iE'}$

$$\left[-\mathcal{G}_0(k,iE_1)\, e^{-iE_1\frac{\tau-\tau_1}{\hbar}}\right]\left[-\mathcal{G}_0(k+q,iE_2)\, e^{-iE_2\frac{\tau_1-\tau_2}{\hbar}}\right]\left[-\mathcal{G}_0(k,iE_3)\, e^{-iE_3\frac{\tau_2}{\hbar}}\right]$$

$$\left[-\mathcal{G}_0(k',iE_4)\, e^{-iE_4\frac{\tau_2-\tau_1}{\hbar}}\right]\left[-\mathcal{G}_0(k'-q,iE_5)\, e^{-iE_5\frac{\tau_1-\tau_2}{\hbar}}\right] e^{iE_n\frac{\tau}{\hbar}}$$

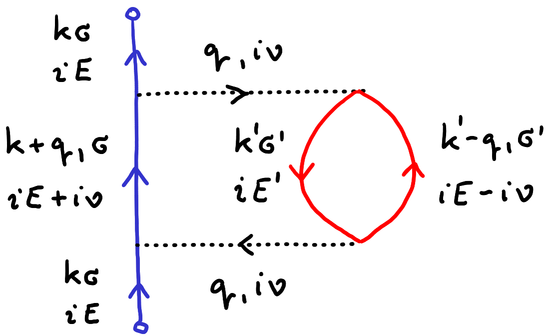


$$\int d\tau \longrightarrow \delta_{iE-iE_1} \qquad \int d\tau_1 \rightarrow \delta_{iE_1+iE_4-iE_2-iE_5} \qquad \int d\tau_2 \rightarrow \delta_{iE_2+iE_5-iE_3-iE_4}$$

$$-\mathcal{G}(k,iE) = (-1) \sum_{qk'\sigma'} \frac{1}{\beta^2} \sum_{iE_1}\sum_{iE_2}\sum_{iE_3}\sum_{iE_4}\sum_{iE_5} (-V_q)^2\, \delta_{iE-iE_1}\,\delta_{iE_1+iE_4-iE_2-iE_5}\,\delta_{iE_2+iE_5-iE_3-iE_4}$$

$$\left[-\mathcal{G}_0(k,iE_1)\right]\left[-\mathcal{G}_0(k+q,iE_2)\right]\left[-\mathcal{G}_0(k,iE_3)\right]\left[-\mathcal{G}_0(k',iE_4)\right]\left[-\mathcal{G}_0(k'-q,iE_5)\right]$$

- Final $\mathcal{G}_0$ labels expressing the "momentum and energy flow"



$i\nu$ ... bosonic Mats. frequency /energy
(difference of two fermionic)

shorthand for $\quad i\nu_m = i\,\frac{2\pi m}{\beta} \quad m \in \mathbb{Z}$

summation over free momenta, spins, and Matsubara energies $\quad q\ k'\ \sigma'\ i\nu\ iE'$

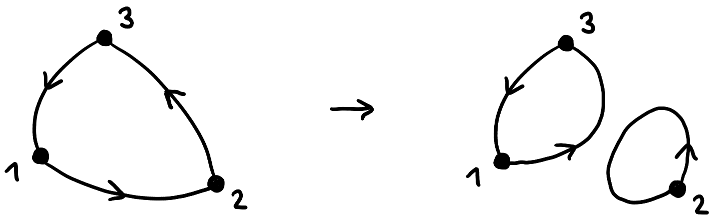
$$-\mathcal{G}(k,iE) = (-1) \sum_{q k' \sigma'} \frac{1}{\beta} \sum_{i\nu} \frac{1}{\beta} \sum_{iE'} (-V_q)^2 \times$$

$$\times \left[-\mathcal{G}_0(k,iE)\right]\left[-\mathcal{G}_0(k+q,iE+i\nu)\right]\left[-\mathcal{G}_0(k,iE)\right]\left[-\mathcal{G}_0(k',iE')\right]\left[-\mathcal{G}_0(k'-q,iE'-i\nu)\right]$$

$$= (-1)(-1)^5 \sum_q (-V_q)^2 \frac{1}{\beta} \sum_{i\nu} \mathcal{G}_0(k,iE)\,\mathcal{G}_0(k+q,iE+i\nu)\,\mathcal{G}_0(k,iE) \times$$

$$\times \sum_{k'\sigma'} \frac{1}{\beta} \sum_{iE'} \mathcal{G}_0(k',iE')\,\mathcal{G}_0(k'-q,iE'-i\nu)$$ $\leftarrow$ Function of $q, i\nu$ only

5 Sign issue – fermionic loops



$c_1 c_3^+ c_3 c_2^+ c_2 c_1^+$

$-G_{13} \; -G_{32} \; -G_{21}$

$c_1 c_3^+ c_3 c_2^+ c_2 c_1^+$

$-G_{13} \; -G_{31} \; +G_{22}$

permutation sign rule:

connect operators with threads

interchange of operators alters sign

$c\,c^+ \; c\,c^+ = -\,c\,c\,c^+\,c^+$

→ count intersections

loop created & sign change observed $\xrightarrow{?}$ rule: each fermionic loop brings an extra -1

AGD

Attention must also be paid to the following. As already mentioned earlier, the sign attaching to each diagram depends on whether the permutation of the Fermi operators ψ is even or not. It is easily seen that a change of sign is connected with the formation of a closed loop in the diagram. The sign of the diagram is therefore determined by the factor $(-1)^F$, where F is the number of closed loops.

Rickayzen

backbone. Now it is not difficult to see that each continuous line makes its own contribution to the sign – the backbone contributes a factor $(-1)^r$ where r is the number of internal vertices on it and each closed loop contributes a factor $(-1)^{s+1}$ where s is the number of internal vertices on it. Hence, the overall factor is $(-1)^{l+2n} = (-1)^l$ where l is the number of closed loops. The overall factor for the

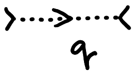
⑥ Dictionary

- propagators

 k, iE $\quad -\mathcal{G}_0(k, iE)$

 k, iE $\quad -\mathcal{G}(k, iE)$

- interactions



$-V_q$

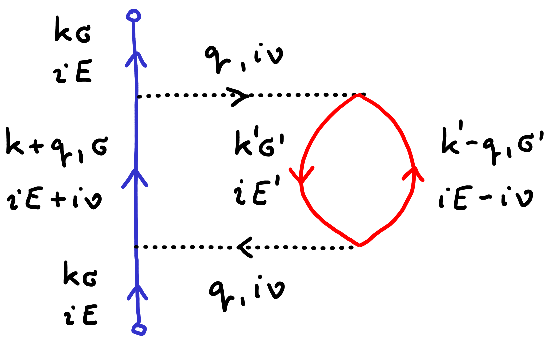
- rules

1) momentum, spin, energy conservation at vertices

2) Free momentum $k \to \sum_k$ $\quad$ Free spin $\sigma \to \sum_\sigma$

Free Matsubara energy $\to \frac{1}{\beta}\sum_{iE_n}$ or $\frac{1}{\beta}\sum_{i\nu_m}$

3) Fermionic loops $\to (-1)^{\#\text{loops}}$



$$-\mathcal{G}(k, iE) = \quad \dots +$$

$$\sum_q V_q^2 \frac{1}{\beta}\sum_{i\nu} \frac{1}{iE-\varepsilon_k} \frac{1}{iE+i\nu-\varepsilon_{k+q}} \frac{1}{iE-\varepsilon_k} \times$$

$$\times \sum_{k'\sigma'} \frac{1}{\beta}\sum_{iE'} \frac{1}{iE'-\varepsilon_{k'}} \frac{1}{iE'-i\nu-\varepsilon_{k'-q}}$$

$$+ \dots$$