The general linearized equations of state for an elastico-viscous liquid can be written in the form[3]:

$$p_{ik} = -pg_{ik} + p'_{ik} \quad (1)$$

$$p'_{ik} = 2\int_{-\infty}^{t} \bar{\psi}\,(t-t')e^{(1)}_{ik}\,(t')\mathrm{d}t' \quad (2)$$

where

$$\bar{\psi}\,(t-t') = \int_0^\infty \frac{N(\tau)}{\tau}\, e^{-(t-t')/\tau}\,\mathrm{d}\tau \quad (3)$$

$N(\tau)$ being the distribution function of relaxation times τ. In these equations, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, p_{ik} is the stress tensor, p an arbitrary isotropic pressure and $e^{(1)}_{ik}$ is the rate-of-strain tensor.

If the equations of state (1) – (3) are substituted into the equations of motion and it is assumed that the physical components of the velocity vector can be written in the form:

$$v_{(r)} = \boldsymbol{R}u(r,\theta)e^{2\pi ift},\ v_{(\theta)} = \boldsymbol{R}v(r,\theta)e^{2\pi ift},\ v_{(\chi)} = \boldsymbol{R}w(r,\theta)\,e^{2\pi ift} \quad (4)$$

where u, v and w are complex, it can be shown that the equation of motion in the χ direction is :

$$-\alpha^2 w = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial w}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial w}{\partial\theta}\right] - \frac{w}{r^2\sin^2\theta} \quad (5)$$

where:

$$\alpha^2 = \frac{-2\pi if\rho}{\displaystyle\int_0^\infty \frac{N(\tau)\mathrm{d}\tau}{(1+2\pi if\tau)}} \quad (6)$$

ρ being the density. In equation (5), second-order terms in the velocities have been neglected. The boundary conditions to be associated with equation (5) are:

$$\left.\begin{array}{ll} w = r\sin\beta\Omega_1 & \text{when } \theta = \beta \\ w = r\Omega_2 & \text{when } \theta = \pi/2 \end{array}\right\} \quad (7)$$

The oscillatory theories associated with viscometers with plane, cylindrical or spherical geometry resolve themselves into finding a solution of an ordinary differential equation[2–5]. If a similar state of affairs is to exist in the problem under consideration a solution of equation (5) must be possible of the form $w = r\sin\theta\Omega(\theta)$. However, no such solution exists, and it is necessary to solve the partial differential equation (5).

The solution of equation (5) presents a problem of some magnitude. It is likely that a general solution can be obtained, but the resulting form for w would certainly be too complicated to be useful to experimentalists who are faced with the problem of interpreting experimental results. In the present article, we shall obtain a solution for the case when α is small.

A Solution of the Equation of Motion. The substitution $w = \partial\psi/\partial\theta$ reduces equation (5) to :

$$\frac{\partial}{\partial\theta}\left[\nabla^2\psi + \alpha^2\psi\right] = 0 \quad (8)$$

where ∇^2 is the Laplacian operator. ψ is arbitrary to the extent of an added function of r and we can choose this arbitrary function such that:

$$\nabla^2\psi + \alpha^2\psi = 0 \quad (9)$$

A solution to (9) which is finite at the origin is given by Lamb[6]:

$$\psi = \sum_{n=1}^{\infty}\left[1 - \frac{\alpha^2r^2}{2(2n+3)} + \frac{\alpha^4r^4}{2.4.(2n+3)\,(2n+5)} - \ldots\right]C_n r^n S_n \quad (10)$$

where r^nS_n is a harmonic function of degree n and the C_n's are constants.

The relevant form for ψ in the present problem is:

$$\begin{aligned}\psi = &\left[r - \frac{\alpha^2r^3}{10} + \frac{\alpha^4r^5}{280} - \frac{\alpha^6r^7}{15,120} + \ldots\right]R_1(\theta) \\ &+ \left[\frac{\alpha^2r^3}{10} - \frac{\alpha^4r^5}{180} + \frac{\alpha^6r^7}{7,920} - \ldots\right]R_3(\theta) \\ &+ \left[\frac{\alpha^4r^5}{504} - \frac{\alpha^6r^7}{13,104} + \ldots\right]R_5(\theta) \\ &+ \left[-\frac{7\alpha^6r^7}{432,432} + \ldots\right]R_7(\theta)\end{aligned} \quad (11)$$

where

$$R_n(\theta) = A_nP_n(\cos\theta) + B_nQ_n(\cos\theta)$$

P_n and Q_n are Legendre functions of the first and second kind, respectively, and A_n and B_n are constants given by:

$$A_n = \frac{\Omega_2\,Q'_n(\beta) - \Omega_1\sin\beta\,Q'_n(\pi/2)}{[Q'_n(\beta)\,P'_n(\pi/2) - Q'_n(\pi/2)\,P'_n(\beta)]} \quad (13)$$

$$B_n = \frac{\Omega_1\sin\beta\,P'_n(\pi/2) - \Omega_2\,P'_n(\beta)}{[Q'_n(\beta)\,P'_n(\pi/2) - Q'_n(\pi/2)\,P'_n(\beta)]} \quad (14)$$

where $P'_n(\pi/2)$ is the value of $\mathrm{d}P_n(\cos\theta)/\mathrm{d}\theta$ at $\theta = \pi/2$, etc.

It will be noticed that the first term of the first series in the expression for ψ satisfies the boundary conditions, but that terms in α^m ($m \geqslant 2$) do not. However, the term in α^2r^3 can be removed by adding to ψ the second series with $n = 3$. The first and second series contain terms in α^4r^5, and these can be removed by the use of a third series with $n = 5$. This process may be continued as far as is desired. In equation 11 terms in α^8r^9 have been neglected.

Equation 11 can be used to develop an oscillatory theory for any given cone and plate viscometer. The theory will be valid for small values of α, provided the gap between the cone and plate is small enough for the neglect of the free surface boundary condition to be justified.

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METALLURGY

The Titanium – Niobium System

THE titanium–niobium system appeared to be very suitable for an examination of the formation and ageing characteristics of martensitic α because published results suggested that an essentially martensitic α structure should be obtained on quenching alloys lying within a fairly wide composition range. A series of titanium–niobium alloys (0–25 at. per cent niobium) was prepared from spectroscopically pure niobium and titanium sponge (as melted hardness 85–90 Brinell hardness number) by tungsten electrode arc melting under 20 cm argon. The 25 g buttons produced were melted twice using a current of 300 amp, and were held molten for at least 1 min during each cycle. The buttons were homogenized in vacuum at 1,000° C for 24 h and then pressed to disks about 0·15 in. thick at temperatures which, according to the published results[1], were within the β field. To reduce contamination the buttons were heated in pure argon prior to pressing, and were exposed to air for the minimum possible time.

Anomalous results obtained on metallographic examination of quenched samples suggested that the published $\beta/\alpha+\beta$ boundary was in error. The position of this boundary was therefore redetermined using resistivity measurements *in vacuo* on specimens 1·5 in. × 0·25 in. × 0·10 in. machined from the forged disks. Up to 20 at. per cent niobium the position of the boundary was clearly marked by a change in slope of the resistivity-temperature curve. The determined boundary temperatures fall below the published values, the discrepancies increasing with increasing niobium content (Table 1).

The transformation to a martensitic phase on quenching from the β field was investigated, using a high-speed thermal arrest apparatus of the type described by Greninger[2]. Using specimens 0·010 in. thick and argon as the quench gas, cooling rates in excess of 1,000° C/sec were obtained. The M_s temperatures determined are in excellent agreement with those reported by Duwez[3] (Table 1).

Table 1

Nb content (at. %)	Temperature of boundary (° C) Hansen *et al.*	Present work	M_s temperature (° C) Duwez	Present work)
0	885	885	855	871
5	810	777	760	758
10	765	645	600	578
15	725	585	400	404
20	680	525	—	212

Table 2

d-obs. (Å)	Intensity	*d*-calc. (Å)	*hkl*
2·644	*m*	2·644	110
2·425	*ms*	2·423	020
2·321	*ms*	2·323	002
2·298	*vs*	2·298	111
2·151	*s*	2·149	021
1·743	*w*	1·745	112
1·677	*s*	1·677	022
1·580	*ms*	1·577	200
1·440	*w*	1·438	130
1·375	*m*	1·374	131
1·336	*ms*	1·336	113
1·322	*w*	1·322	220
etc.			

The structure of the transformed alloys was determined by X-ray diffraction using water-quenched solid specimens, approximately 1 cm across, the surfaces of which had been electropolished. It was at once apparent that the structures of alloys containing more than 5 at. per cent niobium were anomalous. Many of the reflexions from the 'α-phase' were split into two components the separation of which increased with increasing niobium content. The observed diffraction pattern (Table 2) can best be interpreted in terms of a *c*-centred orthorhombic cell, which can be regarded as a simple distortion of the hexagonal cell of α-titanium. The cell dimensions for the 20 at. per cent niobium alloy are $a=3{\cdot}166$ Å, $b=4{\cdot}854$ Å, $c=4{\cdot}652$ Å. The *c*-axis (orthorhombic) corresponds to the *c*-axis of the hexagonal cell while *a* and *b* correspond to the orthogonal axes of the hexagonal cell. The observed axial ratio b/a decreases with increasing niobium content from 1·73 for the true hexagonal cell, to 1·53 at 20 at. per cent niobium.

There is no evidence for segregation of niobium to special sites, and the structure is similar to that of α-uranium (space group C_{mcm}). The (titanium, niobium) atoms are situated in the four-fold position 0, *y*, $\frac{1}{4}$ and preliminary intensity measurements indicate $y=0{\cdot}20$, leading to eight-fold co-ordination around each (titanium, niobium) atom, with four additional neighbours at a slightly greater distance. The small difference in size between titanium and niobium atoms (atomic radius for co-ordination number 12 1·46 *kx* and 1·47 *kx* respectively) is unlikely to account wholly for the distortion to orthorhombic symmetry; it is possible that the distortion is a reflexion of a tendency for niobium to retain eight-fold co-ordination as in the body-centred cubic form. It is possible that a similar phenomenon occurs in other titanium systems such as titanium–aluminium–vanadium in which the effect has been tentatively ascribed to the co-existence of two α phases[4].

Note added in proof. It has since been found that the data on the orthorhombic phase here reported are identical with the little-publicized results of Bagariatskii *et al.* (*Problems in Metallography and Physics of Metals*, **5**, 210; 1958 (in Russian)).

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Interstitial Loops in Neutron-irradiated Molybdenum

NEUTRON irradiation at pile temperatures of the common metals such as copper[1], iron[2] or molybdenum[3] has been found to produce very small prismatic dislocation loops as well as other strain centres on the limits of resolution. Metals subjected to charged particle or fission fragment irradiation, on the other hand, have been found to contain large prismatic loops[4,5] with diameters up to 1000 Å. In these latter experiments, it has been possible to prove that the loops are formed by the aggregation of interstitial atoms. Up to the present this has not proved possible with neutron-irradiated metals as the loops rarely exceed 1 or 200 Å in diameter, which makes application of the diffraction contrast method difficult, if not impossible. It has been inferred from annealing investigations that some of the larger loops (50–200 Å) in neutron-irradiated copper are interstitial, but direct confirmation is not yet available[6]. Experiments have recently been completed in these laboratories which established quite definitely that large dislocation loops, produced by neutron irradiation of molybdenum, are interstitial in nature.

A single crystal of three-pass zone-refined molybdenum was cut into 0·5-mm sections which were then irradiated at 600° C to a total fast flux of 10^{18} nvt. The principal impurities in the crystal are 15 p.p.m. carbon and a total oxygen, nitrogen and hydrogen content of 1 p.p.m. by weight. After irradiation the crystal sections were jet-machined and electrolytically polished to produce specimens for thin foil microscopy. All sections examined contained large prismatic dislocation loops with a density of $2 \times 10^{13}/cm^3$. The loops, which had various angular shapes, were mainly about 1100 Å in diameter. A few loops were as small as 100 Å in diameter and the largest observed was 2000 Å across. Examination of foils prepared parallel to either a (011) plane or a (111) plane showed that the loops tend to lie parallel to {111}. Diffraction contrast experiments established that the loops are virtually pure edge in character with a Burgers vector $a/2<111>$. This result was confirmed when a loop moved out of a foil at room temperature, implying that the loops are glissile.

Tilting experiments carried out in the manner described by Groves and Kelly[7] established that all the loops result from the aggregation of interstitials. More than 30 loops were examined and in all cases the loops were definitely interstitial. The smallest loop examined so far was 500 Å in diameter. A complete account of these experiments and others on the annealing behaviour of the loops and the influence of irradiation on precipitation is being prepared for publication.