

1) a) energijska repre

$$\hat{H} = \hbar\omega \left(\hat{n} + \frac{1}{2}\right) \quad \hat{\rho} = \frac{1}{Z} e^{-\beta \hat{H}}$$

$$\langle n | \hat{\rho} | n' \rangle = \langle n | \sum_{n''} \frac{1}{Z} e^{-\beta \hbar \omega (n'' + \frac{1}{2})} | n'' \rangle \langle n'' | n' \rangle$$

$$= \frac{1}{Z} \sum_{n''} \underbrace{\langle n | e^{-\beta \hbar \omega (n'' + \frac{1}{2})} | n'' \rangle}_{\frac{e^{-\beta \hbar \omega (n'' + \frac{1}{2})}}{e} \delta_{nn''}} \underbrace{\langle n'' | n' \rangle}_{\delta_{n''n'}} = \frac{1}{Z} e^{-\beta \hbar \omega (n + \frac{1}{2})} \delta_{nn'}$$

$$Z = \sum_n e^{-\beta \hbar \omega (n + \frac{1}{2})} = \frac{\frac{\beta \hbar \omega}{e^{\frac{\beta \hbar \omega}{2}}}}{e^{\frac{\beta \hbar \omega}{2}} - 1} = \frac{1}{\frac{\beta \hbar \omega}{e^{\frac{\beta \hbar \omega}{2}}} - e^{-\frac{\beta \hbar \omega}{2}}} = \frac{1}{2 \sinh\left(\frac{\beta \hbar \omega}{2}\right)}$$

$$\hat{\rho}(n, n') = \delta_{nn'} = \frac{1}{Z} e^{-\beta \hbar \omega (n + \frac{1}{2})} \delta_{nn'}$$